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Feasibility and optimality of the initial capital stock in the Ramsey vintage capital model

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ABSTRACT

Using a discrete-time version of the Ramsey Vintage Capital Model we provide a characterization of the set of initial capital stocks compatible with a predefined scrapping time, given the rate of technical progress and the level of capital productivity. Each profile of initial capital stock in that set generates a complete infinite horizon feasible capital path. From that characterization, we prove the existence of a minimum value for the scrapping time of the machines compatible with the rate of technological progress. Moreover, for each level of capital productivity, there exists an upper bound for the technological progress which allows the existence of feasible capital paths with full employment. Finally, we transform the infinite horizon dynamic programming problem into one of finite dimension. We use this to find the optimal lifetime for the machines as well as the optimal composition of the initial capital stocks. A numerical example shows that, in accordance with the infinite horizon approach to the problem, the increase in the rate of technological progress leads to a decrease in optimal scrapping time of capital goods.

1. Introduction

The full employment condition in the economic growth model with heterogeneous capital introduced by Solow et al. (1966) imposes restrictions on the feasible capital paths of the central planner problem. When the technology exhibits perfect complementarity between capital goods and labor force, the condition is even more restrictive due to the absence of substitution between those productive factors. Despite the fact that this condition has been used in the literature to calculate the optimal path and the optimal vintage period length, it has not been explored to analyse the effects of the productivity and the technological progress on the set of feasible initial capital stocks and their corresponding scrapping times. In fact, that condition in addition to the first order condition of the problem, were used by Boucekkine et al. (1997) to characterize the solution of the vintage capital model with linear utility. After that, Boucekkine et al. (1998) extended some of their results for the nonlinear utility case in order to propose a numerical method to solve the problem.

The purpose of this work is to analyse how the restrictions of full employment and the capital market clearing condition exclude some compositions of initial capital stocks in the Ramsey Vintage Capital Model (RVCM). Specifically, we prove that given the productivity and the rate of technological progress, each predefined value for the scrapping time determines the set of feasible initial capital stocks. From this, we prove our main result (Theorem 1) by establishing upper bounds for the rate of technological progress in terms of the capital productivity. Notice that those bounds do not depend on the utility function of the central planner. In the case of additively separable utility, we are able to write the total discounted utility of a feasible capital path in terms of the initial profile of capital stocks of the problem (the capital pre-history). Finally, we find the optimal value for the scrapping time of the machines and the optimal composition of the initial capital stocks. The justification for this analysis is that the long-term optimal capital paths in the RVCM satisfy the labor market clearing condition and have a constant scrapping time (Boucekkine et al., 1997; Hritonenko and Yatsenko, 2008; Malcomson, 1975; Hilten, 1991). Therefore, this work does not analyse the transitional dynamics of the optimal paths but rather its long-term specification. As we will see in Theorem 1 and Section 3, that approach will provide us a close connection between capital productivity and the rate of technological progress (connection not yet explored) as well as a new and simple methodology to find the long-term optimal capital paths.

It is worth noting that the restriction imposed by the full employment condition is innocuous if there is substitution between labor and capital (as in Cooley et al. (1997) and Jovanovic and Yatsenko (2012)). However, most of the works on that model

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suppose perfect complementarity between those two productive factors.

The results have important theoretical and applied implications. In practice, when solving the RVCM, we may be interested in the long-run behavior of the optimal capital path. In that case, our approach provides a simple methodology to find the optimal capital composition in the steady state. From the theoretical point of view, the results allow us to explain why some economies with vintage capital stocks might not obtain sustainable growth without generating unemployment or inefficiently using the installed capital (disequilibrium in the labor market). Thus, the model is capable of explaining unemployment in economies with poor structures of initial capital stocks and high levels of human capital. Reciprocally, it also explains why economies with low levels of human capital and good capital structures must invest in qualification or even import labor force. This is not the first model exhibiting the problems in the labor market triggered by technological progress. In Jovanovic (1998, 2009) it is proved that the indivisibility of the machines and the increase of the technology trade raise the wage inequality in the economy.

This manuscript is divided into four sections. In Section 2 we present the discrete-time version of the RVCM, which is more suitable for the analysis that we are going to perform, and the main result. In Section 3 we show how the central planner problem of the RVCM can be reduced to a finite dimensional problem, where the goal is to find the optimal value for the scrapping time of the machines and as a consequence, the optimal composition of the initial capital stocks. In Section 4 we discuss some conclusions of the work and the proofs are given in the Appendix.

2. Discrete version of the RVCM

In this section we are going to present the discrete version of the RVCM in order to show how the technological progress parameter restricts the initial composition of capital stocks of extended feasible paths. As a consequence we will characterize the set of initial capital stocks compatible with the level of technical progress and with the scrapping time for the machines. In particular, we will show that for each level of technological progress, there is a lower bound for the length period of capital usage that allows for sustainable growth.

Time is discrete and the technology corresponds to that of the standard *AK* model with vintage capital. For each $t \in \mathbb{N} = \{0, 1, \dots\}$, the length period of the vintage, or *scrapping time* of existing machines in t is denoted by $T(t)$ and the parameter representing the Harrod-neutral rate of technological progress is $\gamma \in (0, 1)$. Thus, if $k_j \geq 0$ represents the capital stock (number of machines) in period $j = t - T(t), \dots, t$ to be used in time t, then the amount of labor force needed for those machines to operate is $\gamma^j k_j$. In this setting, a greater technological progress corresponds to a lower value of the parameter γ. Labor has a totally inelastic supply which we normalize to one. There is one representative consumer with instantaneous utility function given by $u : \mathbb{R}_+ \to \mathbb{R}$ and intertemporal discount factor $\beta \in (0, 1)$. Therefore the central planner problem is to find the scrapping time function $T : \mathbb{N} \to \mathbb{N}^+ = \{1, 2, \dots\}$ and paths for the consumption, investment and production $(c_t, k_{t+1}, y_t)_{t\in\mathbb{N}}$ such that:

$$\begin{aligned}
\underset{\{T,(c_t,k_{t+1},y_t)_{t\in\mathbb{N}}\}}{\text{Maximize}} \quad & \sum_{t=0}^{+\infty} \beta^t u(c_t) \\
\text{subject to} \quad & c_t + k_{t+1} = y_t; \\
& y_t = b \sum_{j=t-T(t)}^{t} k_j; \\
& \sum_{j=t-T(t)}^{t} \gamma^j k_j = 1; \qquad t \in \mathbb{N} \\
& k_0, k_{-1}, \dots \text{ given.}
\end{aligned} \tag{1}$$

This is the analogous discrete version of the problem stated in Boucekkine et al. (1997). The restrictions correspond to the market clearing condition in the good market, the *AK* technology defining the total production and the full employment condition respectively. An alternative version of the model, where the technology allows for certain degree of substitution between labor and capital was analyzed by Cooley et al. (1997) and more recently by Jovanovic and Yatsenko (2012). Benhabib and Rustichini (1991) developed a vintage capital model without labor as productive factor. In the rest of this section we are going to analyse the capital paths that satisfy the restrictions of the problem (1).

A *feasible capital path* for (1) is a sequence $\widetilde{\mathbf{k}} = (k_1, k_2, \dots)$ such that there exists a function $T : \mathbb{N} \to \mathbb{N}^+$ which satisfies $k_{t+1} \leq b \sum_{j=t-T(t)}^{t} k_j$ and $\sum_{j=t-T(t)}^{t} \gamma^j k_j = 1$ for all $t \in \mathbb{N}$. The function $T(.)$ is called the *scrapping time function* associated to $\widetilde{\mathbf{k}}$.

Lemma 1 (*Sufficient Condition for Decreasing Scrapping Times*)**.** *If $\widetilde{\mathbf{k}} = (k_1, k_2, \dots)$ is a feasible path for (1) and $k_t > 0$ for all $t \in \mathbb{N}$, then any associated scrapping time function $T(.)$ is a decreasing function.*

Whereas we want to analyse the long-run equilibrium paths that allows for sustainable growth, we will be interested in balanced paths with strictly positive components. Fabbri and Gozzi (2008) and Boucekkine et al. (2005) proved that for either linear or nonlinear utilities, the equilibrium paths for the VCM have strictly positive components. Therefore, Lemma 1 will lead us to consider decreasing scrapping time functions $T(t)$. Since the sequence $\{T(t)\}_{t\in\mathbb{N}}$ has zero as a lower bound, there will exist $t_0 \in \mathbb{N}$ such that $T(t) = T(t_0)$ for all $t > t_0$. Namely, analysis of long-run equilibrium paths should consider constant scrapping time functions. In continuous time versions of the model, the optimality of constant capital lifetime was already proved by Hritonenko and Yatsenko (2008), Boucekkine et al. (1998) and Hilten (1991).

Therefore, for each $T \geq 1$ we will analyse the feasible paths with scrapping time T.

Definition 1. A T-extended feasible path for the problem (1) is a sequence $(k_t)_{t\geq -T}$ such that for all $t \geq 0$:

$$\begin{aligned}
&\text{(i)} \quad k_{t+1} \leq b \sum_{j=t-T}^{t} k_j; \\
&\text{(ii)} \quad \sum_{j=t-T}^{t} \gamma^j k_j = 1.
\end{aligned} \tag{2}$$

Thus, T-extended feasible paths are feasible capital paths where the scrapping times are assumed constants over the whole time horizon. We are restricting ourselves to these paths because we want to analyse the long-run behavior of the solutions of (1).

Condition (ii) in (2), which is the equality between demand and labor supply, implies the *replacement echoes* effect in the model, already used in the literature (Hritonenko and Yatsenko, 2008; Boucekkine et al., 1997). Specifically, if we take the first difference of that equality we will obtain:

$$k_{t+1} = \gamma^{-T-1} k_{t-T}, \quad \text{for all } t \geq 0. \tag{3}$$

Thus, the creative destruction process implies that the capital must grow at rate γ^{-T-1} in each vintage period in order to maintain full employment. This leads us to conclude that, any T-extended feasible path $(k_t)_{t\geq -T}$ satisfies both the replacement echoes effect (3) and the labor market clearing condition (ii) in (2) for its initial capital stock $(k_{-T}, \dots, k_0)$. Reciprocally, it is not difficult to verify that if the initial capital stocks $(k_{-T}, \dots, k_0)$ satisfies $\sum_{j=-T}^{0} \gamma^j k_j = 1$, and the replacement echoes effect (3) is also