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Testing for intertemporal nonseparability[☆]

Ian Crawford[a,b], Matthew Polisson[c,b,*]

[a] Department of Economics, University of Oxford, Manor Road, Oxford, OX1 3UQ, United Kingdom
[b] Institute for Fiscal Studies, 7 Ridgmount Street, London, WC1E 7AE, United Kingdom
[c] Department of Economics, University of Leicester, University Road, Leicester, LE1 7RH, United Kingdom

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ABSTRACT

This paper presents a nonparametric analysis of a common class of intertemporal models of consumer choice that relax consumption independence. Within this class and in the absence of any functional form restrictions on instantaneous preferences, we compare the revealed preference conditions for rational habit formation and rational anticipation. We show that these models are observationally equivalent in the presence of finite data sets composed of prices, interest rates, and consumption choices.

1. Introduction

The discounted utility model is the standard framework for thinking about dynamic consumer behaviour.[1] The model typically supposes that an agent's preferences over consumption profiles can be represented by $\sum_t \beta^{t-1}u(x_t)$, where u denotes a time-invariant, cardinal, and concave instantaneous utility function defined over the period t consumption vector x_t, and where β is the discount factor defined as $1/(1+\rho)$, with ρ denoting the discount rate. A key feature of the discounted utility model is that it explicitly assumes time separability, or consumption independence. This embodies the assumption that an individual's preferences over consumption in any period are independent of consumption in any other period.

That intertemporal separability is a strong and perhaps contentious assumption has of course long been acknowledged—both Samuelson (1952) and Koopmans (1960) recognised it as such. But despite the manifest implausibility of this assumption, it remains popular, mainly because it greatly simplifies the analysis of intertemporal choice.

The two most obvious and straightforward approaches that incorporate intertemporal *nonseparability*, i.e., that allow preferences at a point in time to depend upon consumption choices at others, are rational habit formation and rational anticipation. Rae (1834) was perhaps the first to propose the idea that utility from current consumption can be affected by past consumption. The notion that a knowledge of future consumption can affect present decision making goes back as far as Jevons (1871). Both nonseparable approaches have delivered meaningful insights into consumer behaviour, and both are able to explain empirical consumption 'puzzles' where the time separable benchmark falls short.

Models of habit formation have been developed and applied with some enthusiasm,[2] while models of anticipation have been

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* Corresponding author at: Department of Economics, University of Leicester, University Road, Leicester, LE1 7RH, United Kingdom.

E-mail addresses: ian.crawford@economics.ox.ac.uk (I. Crawford), matthew.polisson@le.ac.uk (M. Polisson).

1 For the origins of this literature, see Rae (1834), Böhm-Bawerk (1889), Fisher (1930), and Samuelson (1937).

2 Contributions in applied microeconomics include Abel (1990), Constantinides (1990), and Campbell and Cochrane (1999) on asset-pricing puzzles, Becker and Murphy (1988) on addiction, and Meghir and Weber (1996) on the identification of intertemporal nonseparabilities and liquidity constraints. Macroeconomic applications include habits as an explanation for movements in asset prices (Boldrin et al., 2001) and the relationship between economic growth and savings (Carroll et al., 2000).

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slower to advance.[3] Nonetheless, the suggestion that anticipation and habit formation may be equally effective in explaining consumer behaviour is at the core of this paper. While habits and anticipation certainly come in many flavours, in general the literature treats them as though they are *distinct*. In the absence of specific parametric restrictions on instantaneous preferences, we show that this is not the case within a common class of intertemporally nonseparable models. We derive the empirical implications of these models in the revealed preference tradition of Samuelson (1948), Houthakker (1950), Afriat (1967), Diewert (1973), and Varian (1982), and demonstrate an equivalence in the presence of finite data sets composed of prices, interest rates, and consumption choices.

The paper is organised as follows. Section 2 introduces the framework within which we investigate the observable implications of intertemporal nonseparability. Section 3 outlines the revealed preference conditions for models of rational habit formation and rational anticipation within this framework. Section 4 contains the main equivalence result of the paper. Section 5 provides some brief concluding remarks.

2. Framework

In order to isolate intertemporal nonseparability, we adhere to many of the principal assumptions of the benchmark discounted utility model—only consumption independence is relaxed.[4] Note therefore that we continue to assume instantaneous preferences that are stable over some horizon, separable aggregation,[5] perfect foresight, exponential discounting, and perfect liquidity.

We let $x_t \in \mathbb{R}^K_+$ denote a vector of consumption goods (where each good is indexed by $k \in \mathcal{K} = \{1, \dots, K\}$) purchased at corresponding prices $p_t \in \mathbb{R}^K_{++}$ in each period of observation $t \in \mathcal{T} = \{1, \dots, T\}$, where $\mathcal{T}$ denotes the set of contiguous periods observed by the econometrician. Discounted prices are given by $\hat{p}_t \in \mathbb{R}^K_{++}$.[6]The econometrician therefore observes a data set of discounted prices and consumption choices $\{(\hat{p}_t, x_t)\}_{t\in\mathcal{T}}$. We make an assumption that these data are in the interior of the consumer's life cycle, i.e., that they are a subset of some larger set of consumption data. Since initial and final conditions are rarely observed in consumption data, this observability assumption is natural and standard. Given these observables, we let $B = \{y_t \in \mathbb{R}^K_+$ for all $t \in \mathcal{T} : \sum_{t\in\mathcal{T}} \hat{p}_t \cdot y_t \le \sum_{t\in\mathcal{T}} \hat{p}_t \cdot x_t\}$ denote the intertemporal budget set, i.e., the set of affordable consumption profiles conditional on observed intertemporal expenditure.

The presence of lags or leads in an intertemporal model typically requires a truncation of the data set. For example, a model with one-period habit formation would normally confine an econometrician to the set of periods $\{2, \dots, T\}$, while a one-period anticipatory effect would restrict her to the set $\{1, \dots, T-1\}$. In what follows, we want to compare models using precisely the same data. We also want to make use of as much data as we have observed. In the preceding example, using all periods of observation would naturally mean that the period 0 lag and period $T+1$ lead of consumption are not observed. In our framework, we merely treat these unobserved consumption vectors as unknown free parameters and ask, as part of the characterisation, whether there exist any out-of-sample consumption bundles such that the observed data are consistent with the model of interest. This allows us both to utilise the entirety of the observed data and also to apply the conditions for both habit-formation and anticipatory effects to the same data. In order to allow for lags and leads of consumption, we make use of two augmented sets of periods which extend beyond the set of observed periods. More specifically, we allow for N lags or leads,[7] and we denote the augmented sets by $\underline{\mathcal{T}} = \{1-N, \dots, T\}$ and $\overline{\mathcal{T}} = \{1, \dots, T+N\}$.

Finally, we ask whether there are necessary and sufficient conditions which guarantee the existence of some instantaneous utility functions $u : \mathbb{R}^{K(N+1)}_+ \to \mathbb{R}$ and $v : \mathbb{R}^{K(N+1)}_+ \to \mathbb{R}$, as well as a discount factor $\beta \in (0, 1]$ and unobserved consumption $x_t \in \mathbb{R}^K_+$ for any $t \notin \mathcal{T}$,[8] such that a consumer could have been solving either

$$\max_{\{x_t\}_{t\in\mathcal{T}}\in B} \sum_{t\in\overline{\mathcal{T}}} \beta^{t-1} u(x_t, x_{t-1}, x_{t-2}, \dots, x_{t-N}) \tag{1}$$

or

$$\max_{\{x_t\}_{t\in\mathcal{T}}\in B} \sum_{t\in\underline{\mathcal{T}}} \beta^{t-1} v(x_t, x_{t+1}, x_{t+2}, \dots, x_{t+N}), \tag{2}$$

where (1) corresponds to rational habit formation and (2) to rational anticipation. We also ask whether the utility functions u and v are necessarily distinct. We formalise this approach in the following section.

3. Revealed preference analysis

3.1. Rational habit formation

We begin by examining the revealed preference conditions for a model of rational habit formation that is standard in the literature.

Definition 1. The data set $\{(\hat{p}_t, x_t)\}_{t\in\mathcal{T}}$ is said to be consistent with rational habit formation if there exist a nonsatiated, continuous, and concave utility function $u : \mathbb{R}^{K(N+1)}_+ \to \mathbb{R}$, a discount factor $\beta \in (0, 1]$, and unobserved consumption $x_t = y_t \in \mathbb{R}^K_+$ for any $t \notin \mathcal{T}$, such that $\sum_{t\in\overline{\mathcal{T}}} \beta^{t-1} u(x_t, \dots, x_{t-N}) \ge \sum_{t\in\overline{\mathcal{T}}} \beta^{t-1} u(y_t, \dots, y_{t-N})$ for all $\{y_t\}_{t\in\mathcal{T}} \in B$.

This definition naturally states that a data set is consistent with rational habit formation if the observed consumption profile delivers weakly greater intertemporal utility than any other affordable profile conditional on some unobserved consumption. In the case of habits, this unobserved consumption is any consumption which lies outside the set of observed periods. While $x_t, y_t \in \mathbb{R}^K_+$ for any $t \notin \mathcal{T}$ are unobserved, we allow them to enter freely subject only to the restriction that $x_t = y_t$ for any $t \notin \mathcal{T}$, i.e., we treat any unobserved consumption as a fixed conditioning variable. We now establish the revealed preference conditions for rational habit formation.

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3 Perhaps most notably, Loewenstein (1987) proposed that instantaneous utility is equal to utility from current consumption plus some function of consumption in future periods. Incorporating future consumption in this way allows the consumer to have a preference for improvements over time and for suffering unpleasant outcomes quickly rather than delaying them. More recently, Caplin and Leahy (2001) have shown that anticipatory utility can explain the equity premium puzzle just as effectively as habit formation.

4 By consumption independence, we mean that instantaneous preferences are allowed to depend upon lags and leads of consumption.

5 As shown in Kubler (2004), the nonseparable representation in Kreps and Porteus (1978) fails to deliver any meaningful empirical content whatsoever unless the intertemporal aggregator is weakly separable. Within our framework, intertemporal aggregation remains additive.

6 Prices are discounted throughout according to $\hat{p}_t = p_t / \prod_{s=1}^{s=t-1}(1 + r_s)$ for all $t \in \mathcal{T} \setminus \{1\}$ and $\hat{p}_1 = p_1$, where $r_t \ge 0$ denotes the rate of interest between period t and $t+1$ for all $t \in \mathcal{T} \setminus \{T\}$.

7 We expect there to be fewer lags or leads than periods of observation.

8 Since an econometrician observes some subset of contiguous periods $\mathcal{T}$ containing consumption data that is in the interior of the consumer's life cycle, then for any $t \notin \mathcal{T}$, consumption is unobserved and therefore enters the characterisation as an existential quantifier.