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Two-period model of insider trading with correlated signals

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ABSTRACT

In this article, we extend the one-period model of Jain and Mirman (1999) for asset trading with two correlated signals to a two period model. We then prove the existence and uniqueness of the Bayesian linear equilibrium. Finally, we perform comparative statics analysis with respect to Kyle (1985). Our findings reveal that adding another correlated signal (the real signal) to the total order flow of Kyle (1985), increases the amount of information incorporated in the stock price at each period and decreases the insider's expected profits at each period.

1. Introduction

Insider trading is said to occur when an individual with special knowledge of a corporation uses this knowledge to buy and/or sell securities such as stocks and bonds to make a profit. This special knowledge is known as material information and includes any pertinent knowledge about a company that is not known to the general public. Suppose, for example, that a company is about to announce that its quarterly earnings were much higher than that which was forecasted in the previous quarter. An individual on the inside of the company, say a manager, who quickly buys up stock in the corporation knowing that the pending announcement will drive up the price of the company's stock, is said to have engaged in insider trading. The underestimated quarterly sales is said to be material information not known to the general public.

Insider trading with long-lived information has been extensively studied in the literature. The seminal work of Kyle (1985) was considered as the benchmark model which studied the strategic dynamic trading and the effect of the microstructure on the informational efficiency of prices. In Kyle (1985), a single asset with random ex post liquidation value and a riskless asset with unitary return, traded among noise traders and a large risk neutral informed trader "the insider" who observed the liquidation value of the risky asset, with the intermediation of competitive makers. The insider acted strategically, i.e., he took into account the effect his demand has on prices.

Kyle (1985) studied the finite horizon (T periods) setting. In each period, the insider submitted his order contingent on his information while the noise traders submitted their aggregated order. The total order flow is then observed by risk neutral market makers and set the market price accordingly.

When all the exogenous random variables are normally distributed, Kyle (1985) solved the dynamic programming problem of the insider and showed that there existed a linear recursive equilibrium which corresponded to a linear Perfect Bayesian Equilibrium of the dynamic game between the insider and the competitive market makers.

Several papers extended the Kyle-type dynamic trading game model. Holden and Subrahmanyam (1992) examined the competition among several risk neutral insiders, all observing the fundamental value of the risky asset. Holden and Subrahmanyam (1994) considered the case when the insiders are risk-averse. In Foster and Viswanathan (1996)'s model, each risk-neutral insider received a noisy signal about the liquidation value. Huddart et al. (2001) studied a version of the Kyle model where the insider has to disclose his trade before the next round of trading. Zhang (2004) characterized the recursive linear equilibrium in the presence of a risk-averse insider with public disclosure.[1] All these papers assumed that at each

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[1] For more details about these models and related works, the reader can consult (Vives, 2010).

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period of the trading game, market makers only observed the corresponding total order flow composed from the insider(s) and the noise traders orders.

Jain and Mirman (1999) were the first to extend the static Kyle-type model by allowing the market maker to observe other than the total order flow signal. Indeed, additional to the total order flow, the market maker in the Jain and Mirman (1999) model observed a correlated signal to the liquidation value of the asset. This new signal took the form of the liquidation value of the asset plus a random noise. Hence, the heteroscedastic and the multiplicative noise of the new signal affected the equilibrium outcomes. Jain and Mirman (1999) proved the existence and uniqueness of a linear equilibrium, i.e. the pricing rule set by the market marker is an affine function of the two signals. They showed that more information is revealed in the stock price and the insider's profits are lower, in the presence of the two signals than in the Kyle model. They also studied the effect of the noise signal on all the equilibrium outcomes.

Several extensions of Jain and Mirman (1999) were studied. For example, Jain and Mirman (2000) studied the effect of insider trading in the presence of both, real and financial markets. Recently, Daher et al. (2014)[2] studied the effect of product differentiation in the real market in the presence of different structures in the financial markets on insider trading. It should be pointed out that Jain and Mirman (1999) and its extensions, examined only the static case.

In this paper we consider the dynamic setting game of Jain and Mirman (1999). Thus, our model extends the dynamic Kyle (1985) model to allow the market maker to observe two correlated signals to the liquidation value at each period of the trading game before he sets the price accordingly. Consequently, the insider, with complete knowledge of the market information structure at each period, has no longer complete market power, as in the Kyle model, to manipulate the stock price and hide the true value of the asset using the liquidity traders orders as camouflage. Indeed, the new signal (referred as the real signal) that the market maker observes at each period, is independent of the insider's order and takes the form of the liquidation plus a noise. Any change in the variance of the real signal's noise, affects the market maker's information regardless of the insider's order. For instance, if the variance of the real signal noise is relatively low, the market maker earns more information about the true value of the asset from the real signal than from the total order flow signal.

This new information structure of the market maker together with the strategic behavior of the insider, leads to several interesting properties of the equilibrium outcomes. For example, when compared to Kyle (1985), we show that the stock price reveals more information in each period. Moreover, the insider's profits at each period are lower in the case of the two signals than in the one signal Kyle model. Finally, the market depth in our model is also affected by the real signal. We show that at each period, the market depth in our model is greater than in the Kyle model.

Moreover, note that solving the dynamic programming problem of the insider is no longer straightforward as in Kyle (1985). Indeed, in Kyle (1985), solving the backward dynamic programming problem of the insider, generated a non parametric cubic equation (see Kyle (1985), Tighe (1989), Holden and Subrahmanyam (1992) and Huddart et al. (2001)). However, adding another signal to the total order flow at each period to the market maker's information set, induces a parametric cubic equation whose solutions are not easily computed. Thus, in order to highlight all the steps of the dynamic setting we will only focus on the two-period model framework.

Noh and Choi (2009) examined an extension version of the dynamic Kyle model, in which the insider has monopoly on two types of information, long-lived and short-lived and thus, additional to the total order flow, the market maker observes at each period some correlated signal about the liquidation value of the asset. Consequently, their model differs from our model in the information structure of the market marker. Nishide (2009) analyzed a Kyle-type continuous-time market model in which liquidity trading is correlated with a noisy public signal that is released continuously. He showed that, the introduction of an additional public signal does not necessarily improve the informational efficiency of the market, depending on the correlation. Caldentey and Stacchetti (2010) extended the Kyle-type model in three different directions. First, the fundamental value of the asset follows a Brownian motion and, therefore, it changes continuously over time. Second, in addition to the initial observation, the insider continuously received a signal of the current fundamental value. Finally, the public announcement is unpredictable.

The paper is structured as follows: in Section 2, we present the two-period model and we characterize the unique linear equilibrium. We then, discuss the properties of this equilibrium. In Section 3, we perform comparative static with respect to the static (Jain and Mirman, 1999) model and to the dynamic (Kyle, 1985).

2. The model

Consider a two period Jain and Mirman (1999) model in which a risky asset with liquidation value $\tilde{z}$,[3] normally distributed with mean p_0 and variance Σ_0, is exchanged in a security market among three types of agents: one risk neutral insider who knows z, the realization of $\tilde{z}$ with certainty before the start of trading, and places at each period n ($n = 1, 2$) an order $\tilde{x}_n$ to buy or sell. Second, liquidity traders whose aggregate order is exogenous and is represented by a random variable $\tilde{u}_n$ ($n = 1, 2$) normally distributed with mean zero and variance σ_u^2. The third type consists of risk neutral market makers who set the stock price in order to clear the market at each period. Following Kyle (1985), the Bertrand competition between market makers in the stock market drives them to set the stock price as the posterior expectation of the liquidation value $\tilde{z}$.

At each period n ($n = 1, 2$) the market makers observe, as in Jain and Mirman (1999), two correlated signals before setting the stock price $\tilde{p}_n$. The first signal is the total order flow $\tilde{r}_n = \tilde{x}_n + \tilde{u}_n$ and the second is a real signal denoted by $\tilde{q}_n = \tilde{z} + \tilde{\varepsilon}_n$ where $\tilde{\varepsilon}_n$ is normally distributed with mean 0 and variance σ_ε^2. Moreover, we assume that $\tilde{z}, \tilde{u}_1, \tilde{u}_2, \tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ are mutually independent. Note that, with two signals, the insider, with complete knowledge of the market marker information structure at each period, does not have complete market power, as in the Kyle model, to manipulate the stock price and hide the true value of the asset using the liquidity traders' orders as camouflage. Indeed, the second or real signal received by the market maker is an independent source of information since it does not depend on the insider's order. Hence, any change in the variance of the real signal, affects the market maker's information regardless of the insider's order. For instance, if the variance of the real signal is relatively low, the market maker acquires more information about the true value of the asset from the real signal than from the total order flow signal.

The insider's trading strategy and the market maker pricing rule are described by the real-valued functions $X = (X_1, X_2)$ and $P = (P_1, P_2)$ such that, $\tilde{x}_n = X_n(\tilde{p}_{n-1}, \tilde{z})$ ($n = 1, 2$), $\tilde{p}_1 = P_1(\tilde{r}_1, \tilde{q}_1)$ and $\tilde{p}_2 = P_2(\tilde{r}_2, \tilde{q}_2, \tilde{r}_1, \tilde{q}_1)$.

[2] See the references within in order to know more about Jain and Mirman (1999) extensions.

[3] Random variables are denoted with a tilde. Realized values lack the tilde. The mean of the random variable is denoted with bar.