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Finite horizon consumption and portfolio decisions with stochastic hyperbolic discounting[☆]

Ziran Zou[a], Shou Chen[a,*], Lei Wedge[b]

[a] Business School of Hunan University, Changsha, Hunan 410082, China

[b] College of Business, University of South Florida, Tampa, FL 33620, USA

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ABSTRACT

We study finite horizon consumption and portfolio decisions of time-inconsistent individuals by incorporating the stochastic hyperbolic preferences of Harris and Laibson (2013) into the classical model of Merton (1969, 1971) with constant relative risk aversion (CRRA). We obtain closed-form solutions for optimal consumption and portfolio choices for sophisticated individuals with log utility and numerical solutions for those with power utility. Compared to the results of Merton, we find that stochastic hyperbolic discounting increases the consumption rate but has no effect on the share of wealth invested in the risky asset.

1. Introduction

Most economic decisions are intertemporal in nature and involve tradeoffs between current and future rewards. The theory of discounted utility has become the standard framework in economics for analyzing intertemporal choices. An important ingredient of the theory is the discount function that discounts delayed rewards to the present for decision making. The exponential discount function with a constant discount rate has been the most widely used discount function in the literature. According to Strotz (1955), it is also the only discount function that leads to time-consistent preferences, where an individual has no incentives to deviate from an ex ante optimal plan in future times.

Overwhelming evidence has been documented in psychology and behavioral science that time inconsistency is standard in human preferences (see e.g., Thaler and Shefrin, 1981; Ainslie and Herrnstein, 1981; Ainslie and Haslam, 1992; Loewenstein and Prelec, 1992; Kirby and Herrnstein, 1995; Myerson and Green, 1995; McClure et al., 2004; DellaVigna and Malmendier, 2006). That is, in pursuing immediate gratification, individuals often exhibit a reversal of preferences when choosing between a smaller, earlier reward and an alternative larger, but later reward. The hyperbolic discounting model has become the most widely accepted framework for modeling time-inconsistent preferences in economics.[1] Prelec (2004) argues that "Few economic hypotheses have advanced so rapidly from the fringe to the mainstream as hyperbolic discounting". A huge literature has been developed to address a wide range of issues in economics based on hyperbolic discounting, which includes Barro (1999), O'Donoghue and Rabin (1999), DellaVigna and Malmendier (2004), Grenadier and Wang (2007), and Palacios-Huerta and Pérez-Kakabadse (2013), among others.

Individuals with time-inconsistent preferences are regarded as naive or sophisticated depending on whether they realize that their preferences will change in the future.[2] Naive individuals assume future selves will act in the interest of the current self and make decisions without considering future selves' true preferences.

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* Corresponding author. Tel.: +86 731 88823895.
E-mail address: chenshou@hnu.edu.cn (S. Chen).

1 Gul and Pesendorfer (2001) propose an alternative approach to model time-consistent preferences by suggesting that temptation but not preference change might be the cause of dynamic inconsistent behavior. Miao (2008) adopts the Gul–Pesendorfer approach to solve the problem of optimal option exercise by dynamic programming.

2 The distinction between naivety and sophistication, first proposed by Strotz (1955), has been analyzed by Akerlof (1991) and O'Donoghue and Rabin (1999), among others.

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Because of the changing preferences, the plan of naive individuals is not time-consistent and cannot be implemented in practice. On the other hand, sophisticated individuals choose a time-consistent plan that is optimal even given the anticipated actions that will be taken by future selves according to their changing preferences.

In this paper, we study the intertemporal consumption–savings and portfolio-selection problem, a fundamental issue in modern economics and finance, for sophisticated individuals with time-inconsistent preferences and a finite lifetime. Since the seminal works of Merton (1969, 1971), almost all existing studies on the intertemporal consumption and portfolio problem assume exponential discounting with constant discount rate and thus imply time-consistent preferences. We extend the current literature by incorporating the stochastic hyperbolic discounting model of Harris and Laibson (2013) into Merton's classical framework.

Our paper makes several important contributions to the literature. First, we derive the Hamilton–Jacobi–Bellman (hereafter HJB) equation of optimal consumption and portfolio choices for sophisticated individuals with finite investment horizon using the dynamic programming approach of Karp (2007). Second, we obtain closed-form solutions for the log utility function and numerical solutions for the power utility function. Third, we study the impact of stochastic hyperbolic discounting on the dynamic behaviors of expected wealth and consumption and expected lifetime discounted utility. Compared to the results of Merton, we find that stochastic hyperbolic discounting increases the consumption rate but has no effect on the share of wealth invested in the risky asset.

Our paper complements several recent studies on similar issues in the literature. For example, Marín-Solano and Navas (2010) study the consumption and portfolio rules when the discount function is deterministic with a decreasing discount rate and obtain closed-form solutions for naive and sophisticated individuals. Gong et al. (2007) and Palacios-Huerta and Pérez-Kakabadse (2013) study the optimal consumption and portfolio rules for sophisticated individuals with infinite horizon and stochastic hyperbolic discounting. While their approach cannot be applied to the finite horizon case, we can easily extend our analysis to obtain optimal solutions for the infinite horizon case.

The rest of the paper is organized as follows. Section 2 introduces the basic model setup. Section 3 derives the HJB equation for sophisticated individuals with stochastic hyperbolic discounting. Section 4 considers two special cases of log and power utility. Section 5 compares the dynamic behaviors of expected wealth and consumption of individuals with instantaneous gratification and exponential discounting. Section 6 develops numerical solutions for power utility. Section 7 concludes and the Appendix provides technical details.

2. Model setup

In this section, we introduce the basic modeling framework of Merton's consumption and portfolio problem. We also discuss the stochastic hyperbolic discounting preferences of Harris and Laibson (2013).

2.1. The consumption and portfolio problem

Consider an individual facing the intertemporal consumption and portfolio problem of Merton (1969, 1971). Suppose the individual's wealth $w(t)$ at any time t can be invested into two assets: a risk free asset that pays a rate of return r with certainty, and a risky asset whose price follows geometric Brownian motion:

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma dz_t, \tag{1}$$

where μ and σ represent the mean and volatility of the asset return respectively, and z_t is a standard Wiener process. Following Merton (1969, 1971), we assume a complete market with no borrowing constraints.

At any time t, the individual needs to choose $c(t)$, an instantaneous rate of consumption, and $\alpha(t)$, the share of wealth invested in the risky asset. Assuming that the individual has no wage income, then the change in the individual's wealth during a small time interval dt is equal to the difference between the investment proceeds and consumption,

$$dw(t) = [\alpha(t)(\mu - r)w(t) + rw(t) - c(t)]dt + \sigma\alpha(t)w(t)dz_t, \tag{2}$$

with the initial condition w_0.

The individual needs to choose a time-consistent consumption and portfolio policy to maximize the following expected discounted utility of consumption over a finite and an infinite planning horizon, respectively:

$$\max_{\alpha(s),c(s)} E\left[\int_t^T D(t,s)u(c(s))ds + D(t,T)F(T,w(T))\right], \tag{3}$$

$$\max_{\alpha(s),c(s)} E\left[\int_t^\infty D(t,s)u(c(s))ds\right], \tag{4}$$

where $u(c(s))$ is the utility function, $D(t,s)$ denotes the discount function that discounts the utility of consumption at s to the present time t, and $F(T, w(T))$ is the bequest function.

2.2. Stochastic hyperbolic discounting

To study the consumption and portfolio problem of an individual with time-inconsistent preferences, we incorporate the stochastic hyperbolic discounting function of Harris and Laibson (2013) into Merton's framework. As in Harris and Laibson (2013), the discount interval is divided into two subintervals: the present interval and the future interval. Payoffs in the present interval are discounted exponentially with a constant discount rate ρ, whereas payoffs in the future interval are first discounted exponentially with ρ and then further discounted by an additional factor β, where $0 < \beta \le 1$. Thus the discount function $D(t,s)$ can be expressed as

$$D(t,s) = \begin{cases} e^{-\rho(s-t)}, & s \in [t, t+\tau); \\ \beta e^{-\rho(s-t)}, & s \in [t+\tau, \infty), \end{cases} \tag{5}$$

where $[t, t+\tau)$ is the present interval and $[t+\tau, \infty)$ is the future interval. The duration of present interval τ is stochastic and exponentially distributed with parameter λ. The stochastic hyperbolic discount function $D(t,s)$ satisfies the assumption of stationarity, i.e., $D(t, t+s) = D(0,s)$. The expected duration of the present interval is $E[\tau] = \frac{1}{\lambda}$. The smaller the λ, the larger the expected duration of the present interval. When $\lambda = 0$, the duration of the present interval is ∞, which means that the discount function degenerates to an exponential discount function with the constant discount rate ρ. The parameter β $(0 < \beta \le 1)$ reflects the degree of the present bias of the preferences. The smaller the β, the larger the present bias. When $\beta = 1$, there is no difference between the present and the future interval, which implies that the discount function $D(t,s)$ again degenerates to an exponential discount function with the constant discount rate ρ.

The stochastic hyperbolic discount function $D(t,s)$ implies that the individual's preferences change over time. When the decision time goes from t to t', $t' > t$, the marginal rate of substitution of utility at s for s' changes from $\frac{D(0,s-t)}{D(0,s'-t)}$ to $\frac{D(0,s-t')}{D(0,s'-t')}$.[3] It is easy to see

[3] When the decision point is t, the value of one utility received at the future time s is $D(t,s)$. The stochastic variable $D(t,s)$ satisfies the assumption of stationarity. Therefore, at time t, the marginal rate of substitution of utility at s for s' is $\frac{D(t,s)}{D(t,s')} = \frac{D(0,s-t)}{D(0,s'-t)}$.