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Deadlines in stochastic contests

Matthias Lang[a], Christian Seel[b,*], Philipp Strack[c]

[a] Department of Economics, Humboldt-Universität zu Berlin, Germany
[b] Department of Economics, Maastricht University, The Netherlands
[c] Microsoft Research, United States

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ABSTRACT

We consider a two-player contest model in which breakthroughs arrive according to privately observed Poisson processes. Each player's process continues as long as she exerts costly effort. The player who collects the most breakthroughs until a predetermined deadline wins a prize.

We derive Nash equilibria of the game depending on the deadline. For short deadlines, there is a unique equilibrium in which players use identical cutoff strategies, i.e., they continue until they have a certain number of successes. If the deadline is long enough, the symmetric equilibrium distribution of an all-pay auction is an equilibrium distribution over successes in the contest. Expected efforts may be maximal for a short or intermediate deadline.

1. Introduction

There is a large amount of literature on all-pay auctions that are often informally motivated as reduced-form models of a contest.[1] The bid in the auction serves as a proxy for the effort or production cost each participant incurs in the contest; think, for example, of an R&D, job promotion, or procurement contest. This paper formally raises the question how all-pay auctions relate to stochastic contests.

For this purpose, we consider a contest with two main distinctive features. First, each player observes her own progress over time, but has no information about the progress of her rivals. Contrary to a silent timing game, however, each player's progress is stochastic. Second, the contest success function has a cumulative structure, i.e., separate steps add up to the final success.

Many procurement and R&D contests feature these properties: typically contestants conduct their research in secrecy and there are several steps in the development of a jet fighter or a vaccine, for example. But the model also accommodates other contests such as grant competitions: every contestant writes a grant application without knowing the quality of the other applications. Spending more time on the application might increase its quality, and thereby the chance of receiving the grant.

Although most real-world contests and theoretical definitions of a contest contain a deadline, most of the literature abstracts from the deadline by either analyzing static models or setting the deadline to infinity. This paper explicitly focuses on the influence of the deadline on the Nash equilibria of the contest. More precisely, we identify equilibria of the contest for different deadlines and compare them.

Formally, we consider a stochastic contest model in which players decide when to stop privately observed Poisson processes. As long as a player does not stop his process, successes arrive according to a Poisson process. The player who accumulates most Poisson arrivals until the contest deadline wins a prize. Ties are broken randomly. Players maximize expected profits.

The analysis proceeds as follows. First, we recall the bidding distribution in a Nash equilibrium of an all-pay auction in which the set of bids is countable as a benchmark. Under a genericity assumption, we show in Proposition 1 that this distribution is the unique symmetric equilibrium distribution of the auction.

A stopping strategy in the contest induces a probability distribution over the number of successes at the stopping time. Proposition 2 derives a time bound above which the symmetric Nash equilibrium distribution of a discrete all-pay auction is an equilibrium distribution of the contest. By Proposition 3, the equilibrium set of the all-pay auction is identical to the equilibrium set of the contest for an infinite deadline. Hence, the all-pay auction provides a suitable model of a contest which lasts sufficiently long.

* Correspondence to: Maastricht University, School of Business and Economics, 6211 LM, Maastricht, The Netherlands. Tel.: +31 0433883651.
E-mail addresses: lang@uni-bonn.de (M. Lang), c.seel@maastrichtuniversity.nl (C. Seel), philipp.strack@gmail.com (P. Strack).

1 For example, Hillman and Samet (1987), Hillman and Riley (1989), Baye et al. (1996), Che and Gale (1998), Siegel (2009), and Bos (2012) study all-pay auctions with a continuous bid space, while Dechenaux et al. (2003, 2006), and Cohen and Sela (2007) scrutinize all-pay auctions in which the set of bids is countable.

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Table 1
Summary of our equilibrium characterization.

Deadline	Equilibrium type	Characterization	Binding deadline
Short	Cutoff	Unique equilibrium	Yes
Intermediate	Time-dependent cutoff	Example	Yes
Long	All-pay auction	An equilibrium	No
∞	All-pay auction	Equilibrium equivalence	No

For short and intermediate deadlines, it is impossible to replicate the equilibrium distribution of the all-pay auction in the contest. For short deadlines, Propositions 4 and 5 uniquely characterize equilibrium strategies as cutoff strategies: players only stop if they reach a certain number of successes or the game ends. Intuitively, they strictly prefer to continue for all low values. Yet, they strictly prefer to stop above some cutoff value, since the probability that the rival reaches this value until the deadline is too low to make continuation profitable.

Proposition 6 shows that – depending on the parameters – there might be an intermediate region for which neither a cutoff equilibrium, nor an all-pay auction equilibrium exists. In this case, we obtain equilibria in which strategies depend both on the remaining time and the number of successes. We discuss the properties of such an equilibrium in an example.

We define a deadline to be *binding* for an equilibrium if a player could increase her payoff by unilaterally postponing the deadline. Proposition 7 shows that the deadline is non-binding in the all-pay auction type equilibrium and binding in any other equilibrium. Intuitively, only in the all-pay auction equilibrium, each player is indifferent whether to stop at any point of her support at any point in time. Thus, no player would benefit from a longer deadline.

Table 1 briefly summarizes our results for different deadlines.

Finally, we compare expected equilibrium efforts for different deadlines. We discuss an example to show that the equilibrium effort for a short deadline could be higher than for a longer deadline. Intuitively, a shorter deadline reduces the possible competition time, but might increase the intensity of competition in the remaining time. Hence, a contest designer who wants to maximize equilibrium efforts is sometimes better off with a short deadline. In this case, the equilibrium of the contest differs from that of an all-pay auction which illustrates a potential drawback of analyzing all-pay models as a shortcut for contests.

In the literature, there are surprisingly few multi-period contest models in which each player's decision problem is dynamic. The most prominent model in this class in which there is no interaction between the players over time is Taylor (1995). He analyzes a T-period model. The technology in Taylor (1995) is quite different from this paper: in each period, each player decides whether she wants to take an additional draw. A draw is a realization of a random variable. The distribution of the random variable is commonly known and identical across players and time periods. At the deadline T, the player with the highest overall draw wins. In equilibrium, each player stops only if he has a draw above a certain cutoff. Equilibrium efforts are monotonically increasing in the deadline.

Seel and Strack (2013) consider a contest model with the same information structure as in the present paper without a deadline. In their framework, players have to stop a Brownian motion with a drift. They bear no costs for a later stopping time, but they have to stop if their process hits zero. In equilibrium, players do not stop immediately even if the drift is negative.

We proceed as follows. In Section 2, we set up the model. Section 3 contains the main results of the paper. Section 4 discusses the results and concludes. Proofs not provided in the main text are relegated to the Appendix.

2. The contest model

Consider a contest with a fixed deadline $T < \infty$ and two players. At every point $t \le T$, each player $i \in \{1, 2\}$ privately observes a time-homogeneous Poisson process X_t^i with intensity $\lambda \in \mathbb{R}_+$ and jump size 1. A strategy of player i is a stopping time $\tau^i \le T$ with respect to the natural filtration $\mathcal{F}_t^i$ generated by the process X_t^i. The stopping time induces a probability distribution over values of the process at the stopping point. We denote this distribution by $F^i: \mathbb{N}_0 \to [0, 1]$, where $F^i(x) = \mathbb{P}(X_{\tau^i}^i \le x)$. The associated probability measure is denoted by $f^i(x)$. Stopping at time t entails costs of $c\,t$.

To interpret the stopping decision, think about a player i who chooses an effort level $\eta_t^i \in \{0, 1\}$ at any time $t \le T$. The effort decision $\eta_t^i = 1$ indicates continuing to work on a project at a flow cost of effort of c, while $\eta_t^i = 0$ indicates not working on the project anymore. In this model, the arrival rate of the stochastic process X_t^i at time t equals $\lambda \eta_t^i$. This formulation of the problem in terms of efforts is mathematically equivalent to our formulation in terms of stopping times as $\int \eta_t^i dt$ equals the realization of the stopping time τ^i. Hence, we sometimes refer to the stopping time τ^i as a player's effort.

The player who has more Poisson arrivals at her stopping time τ^i wins a prize p. Ties are broken randomly. Thus, each player's payoff is

$$\pi^i = \begin{cases} p - c\tau^i & \text{if } X_{\tau^i}^i > X_{\tau^j}^j \\ \dfrac{p}{2} - c\tau^i & \text{if } X_{\tau^i}^i = X_{\tau^j}^j \\ -c\tau^i & \text{otherwise.} \end{cases}$$

Players maximize expected payoffs. Define the payoff process $(\Pi_t^i)_{t \in \mathbb{R}_+}$ of player i as his expected payoff of stopping immediately, i.e.,

$$\begin{aligned} \Pi_t^i &= \mathbb{E}(\pi^i \mid X_t^i, \tau^i = t) \\ &= p\,\mathbb{P}(X_t^i > X_{\tau^j}^j) + \frac{p}{2}\mathbb{P}(X_t^i = X_{\tau^j}^j) - ct. \end{aligned} \tag{1}$$

3. Equilibrium analysis

In this section, we derive Nash equilibria depending on the deadline. We use Nash equilibrium as the solution concept since no player receives any information about his rival over time.

3.1. Long deadlines

As a benchmark, we first consider the Nash equilibrium of a related static model, the all-pay auction with discrete bids. We then show that the symmetric equilibrium distribution of the all-pay auction is also an equilibrium distribution in the contest and construct an explicit stopping time which leads to this distribution.

3.1.1. The all-pay auction

Consider a model with two risk-neutral players indexed by $i = 1, 2$. A pure strategy of player i is a bid $x^i \in \mathbb{N}_0$. A mixed strategy of player i is a probability measure $f^i: \mathbb{N}_0 \mapsto [0, 1]$. Denote the associated cumulative distribution function by $F^i(z) = \mathbb{P}(x^i \le z) = \sum_{y=0}^{z} f^i(y)$. The agent with the highest bid wins a prize $\hat{p} \in \mathbb{R}_+$ and both players pay their bids x^i. Ties are broken randomly. Hence, the profit of player i from bidding x^i is

$$u^i(x^i, x^j) = \begin{cases} \hat{p} - x^i & \text{if } x^i > x^j \\ \dfrac{\hat{p}}{2} - x^i & \text{if } x^i = x^j \\ -x^i & \text{otherwise.} \end{cases}$$