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Viscous damping on flexural mechanical resonators

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ABSTRACT

A new analytical formula predicting the quality factor of a wide range of flexural mechanical resonators operating in a viscous fluid medium is presented. The formula is derived from a fluid analogy used in the single beam Euler–Bernoulli bending theory, and an extension to tuning forks is proposed. Comparisons with experimental data extracted from the literature are presented. New experiments have also been carried out on different kinds of resonators to investigate the pressure dependency of their quality factors in air. In both cases, very good agreement is obtained for resonator sizes ranging from micrometric to macroscopic sizes.

1. Introduction

Flexural mechanical resonators operated in vacuum have been extensively used in the past century for various applications, taking the most of their sharp resonance characteristics and their relative small size.

These resonators have been widely used for time and frequency applications [1], especially since the invention of the electronic watch based on high quality factor quartz tuning forks [2].

They have also been used as sensing elements for miniature vibrating inertial sensors [3], mainly since the 1980s. These micro electromechanical systems (MEMS) known as Coriolis Vibrating Gyrometers (CVG) [4–6] and Vibrating Beam Accelerometers (VBA) [4,7] have been designed for accelerations and rotations ultra-sensitive measurements using a capacitive or piezoelectric method of detection. Accelerations are deduced from a shift of their resonance frequency which is continuously tracked. Rotations are deduced from the measurement of the Coriolis force applied to such resonators primarily and constantly excited.

Clamped–Clamped beams have been used for example as pressure sensors [8]. The pressure to be measured stresses a membrane bounded to one of the beam's anchor, which in turn shifts the resonance frequency of the beam.

Flexural mechanical cantilevers have been used as probes in the atomic force microscopy domain [9]. In tapping mode of operation, their sharp resonance is the key for accurate tip-to-sample distance measurement.

Several applications of flexural MEMS cantilevers as magnetic field sensors have been reported [10,11]. These MEMS are covered by a ferromagnetic material which interacts with an external magnetic field, hence modifying their resonance frequency.

More recently, the use of nano electromechanical systems (NEMS) for weigh applications also appeared, and already achieved mass measurements below the zeptogram scale under vacuum [12,13].

Some applications however require a complete immersion of the resonant structure inside a fluid medium. It is for example used for highly sensitive chemical sensing platforms (microbalances). Masses below the attogram have already been detected in air at atmospheric pressures [12]. Contact force measurements using tuning forks also have to be performed in situ. We can for example cite photo acoustic force detections [14], resonant optothermoacoustic detection [15] or static pressure measurements. Temperature measurements within a 0.001 °C precision can also be achieved through the dependence of the resonance frequency to the temperature [16].

In any case, the surrounding fluid strongly modifies the characteristics of the resonance, and hence can dramatically reduce the sensors performance compared to vacuum. An effective tool for modeling such reduced characteristics is hence crucial for any optimization or prediction purpose. In the following, we will restrain to the common case of resonators made of beams with a circular or rectangular cross section. We also focus on viscous damping, which is most of the time the dominant damping source[19].

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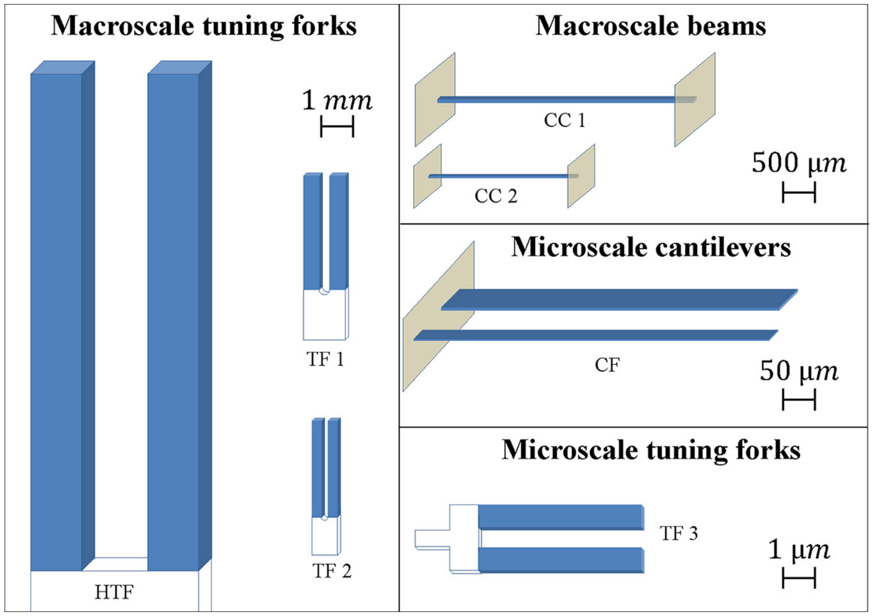

Fig. 1. Aspect comparison of flexural mechanical resonators used to carry out experimental comparisons detailed in Section 4, illustrating various geometrical sizes among existing resonators. HTF is a large in-house quartz tuning fork operated on its out-of-plane first flexural mode, whereas TF 1 and TF 2 are commercially available quartz tuning forks operated in their in-plane first flexural symmetric mode. TF 3 is a diamond tuning fork reported in the literature [17]. CC 1 and CC 2 refer to in-house clamped–clamped beams, and CF refer to typical cantilevers reported in the literature [18]. Exact dimensions are listed in Table 2.

2. Viscous damping state of the art

How does viscous damping exerted by a fluid affect the bending of a beam?

To the best of our knowledge, Newell [20] is the first to give an analytical expression for a beam immersed in an unbounded viscous fluid. He chose to apply the very well-known Stokes' expression for the hydrodynamic force per unit area applied by any fluid on a structure. This approach is extremely simple since neither the thickness of the vibrating structure nor the exact geometry is taken into account. The presence of a nearby static structure is also excluded.

Later on, Tang et al. [21] proposed a model to estimate the quality factor of laterally driven resonant structures close to a static structure. Even if the oscillating plates considered are not flexural mechanical resonators strictly speaking, they are still free–free beams oscillating in a fluid medium and hence perfectly suit the problem. Couette flow underneath the plate is found to be the dominant dissipative process when the distance between the oscillating plate and the surface is sufficiently small. Observing large overestimations of the quality factor predicted by the previous model, air drag on the top surface is introduced [22] and is assumed to behaves as a Stokes flow. An empirical formula based on the superposition principle is presented, mainly because the front air resistance description and edge effects were only accessible through experiment. All of these first attempts were nevertheless not sufficient to correctly describe fluid damping on the constantly increasing resonators' diversity.

The rigorous fluid mechanics problem constitutes a formidable challenge and neither simple analytical formula nor universal model has yet appeared in the literature. For the past 20 years, two very different kind of approaches have been applied to improve viscous damping models. The first approach tends to solve the full problem analytically under limited approximations, by inserting the fluid forces into the Euler–Bernoulli bending theory. It started with Sader [23] and has been constantly refined since then [24–27]. Although a general numerical method is proposed, no simple expression can be extracted for resonators with an arbitrary cross section. The case of infinitely thin rectangular cantilevers on any flexural mode number however has a complex analytical solution and as well as a rather straightforward numerically approximated solution. The rigorous model developed is also able to take into account a static surface nearby or the compressibility of the fluid, although the two latter effects can only be apprehended through advanced numerical simulations. The Euler–Bernoulli bending formalism has also been used for laterally driven resonators by Cox et al. [28], and also deals with infinitely thin cantilevers immersed in an unbounded fluid. The crucial point is the use of Stokes' fluid damping force per unit area. The analytical work assumes that the edge and pressure effects acting on the thickness are negligible compared to the shear damping force.

The second approach tries to establish analogies in order to get an approximate but simple analytical formula. The pioneering work of Hosaka [29] established an analogy between a flexural mechanical cantilever and a string of spheres. Indeed, the exact solution of an oscillating sphere immerged in an unbounded fluid exists and is very simple [30]. Hosaka used this well-known result to express a new simple expression of the quality factor. However, the solution only applies to the first flexural mode of a very thin cantilever. The discrepancies between theory and experiment can also be sometimes substantial, especially for macroscopic size resonators. Using the same analogy, Vignola [17] established a refined formula by adjusting the effective surface used by Hosaka.

In the past 10 years, increments applied on both approaches have mostly tried to generalize the solutions using numerical simulations. Lee extended this way the work of Hosaka for micro cantilevers [31], leading to a more reliable result although limited in its validity to some particular dimensions and experimental conditions. Cox also extended his own results for laterally driven micro resonators with numerical simulations [32] and derived a more accurate expression for the quality factor of any beam with a rectangular cross section.

The method used in this paper lies between the two previously described approaches. We include a new analogy in the rigorous