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# The Samuelson condition and the Lindahl scheme in networks

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We study optimal provision of public goods in the network, showing that the Samuelson condition and the Lindahl scheme can both be substantially extended to characterize the expenditure on such public goods. Couched in terms of the structure of the network, the extended Samuelson condition and Lindahl scheme formulate precisely how the local publicness of a local public good fundamentally determines its optimal provision and the personalized price that the Lindahl tax-payer faces. Independence of the optimal allocation from the distribution of resources at interior Samuelsonian solutions in the network generally fails to hold even for the Bergstrom-Cornes preferences.

## 1. Introduction

The past one and a half decades have seen impressive progress in analysis of Nash games of the private provision of public goods in networks (refer to e.g. Goyal and Moraga-Gonzalez, 2001, Bramoullé and Kranton, 2007 and Allouch, 2015). In contrast, cooperative Lindahl-Samuelson solutions for optimal expenditure and taxation on public goods in networks have not yet received much attention, let alone serious treatment, in the burgeoning literature of network economics.[1] This paper aims to help fill this remarkable void, framing local public goods as public goods in networks and thereby extending the Samuelson condition and the Lindahl scheme to accommodate more general forms of public goods. As such, it might also serve as a useful reminder of the great potential presented by the study of cooperative games in networks.

In the literature on the Samuelsonian public good, the public good is generally assumed to be global, i.e., anyone's contribution to the public good inclusively and equally benefits all members of the economy. But this postulate appears too restrictive. The well known Marshallian knowledge spillover in an industrial cluster is typically confined to a rather limited set of firms; that is, innovation diffusion is often local and hence dictated in great measure by social and/or geographical networks in the industry (see, e.g., Jaffe et al., 1993). Such examples of *local* public goods abound indeed, widely ranging from R & D activities (Goyal and Moraga-Gonzalez, 2001; Andersson and Ejermo, 2005), localized information sharing in the labor market (Granovetter, 1995; Calvó-Armengol and Jackson, 2004), to supply of public goods in ethnic communities (Dasgupta and Kanbur, 2003).

For such economies of a local public good, a good deal of analysis has been conducted into the non-cooperative games of its private provision (see, Bramoullé and Kranton, 2007; Sun, 2012; Allouch, 2015). In this study, we turn to the cooperative Samuelson-Lindahl solutions to the expenditure on and financing of the local public good by framing it as a public good in an appropriately defined network, wherein the public good provided by any agent is non-excludable and non-rivalrous only along links in the network (technical details are to be found in the next section). We show that the Samuelson condition and the Lindahl scheme can both be substantially generalized to characterize the collective expenditure on such public goods. Couched in terms of the structure of the network, the extended Samuelson condition and the Lindahl tax enable us to formulate accurately how the local publicness of a local public good fundamentally determines its optimal provision and the personalized price that the Lindahl tax-payer faces.

The study of the Samuelson-Lindahl solution to optimal expenditure on and financing of local public goods has important bearings. It is worthwhile to note that the key channel through which the network shapes the socioeconomic behavior of individuals is to localize the socioeconomic interaction among them. Such localization effectively serves as a powerful channel to foster social cooperation, not least because it renders fragmentation of the community into a number of smaller sub-communities each of which tends to be more cooperative (for empirical evidence that cooperation is more often found in smaller

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[1] Jackson's (2009) and Galeotti et al.'s (2010) analytical reviews of network games, and Shy's (2011) survey of economic analyses of network effects, arguably the most influential reviews and surveys of the literature, all focus on Nash games in the network.

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communities, refer to, Bardhan, 2000, Fujiie et al., 2005, and Bandiera et al., 2005 among many others). Moreover, networks are more likely to emerge among more homogenous individuals, a phenomenon that is referred to in the literature of network economics as homophily wherein cooperation, especially in providing and sharing information and other local public goods, is promoted by homogeneity (McPherson et al., 2001; Banerjee et al., 2013).[2]

One potentially fruitful application of the extended Samuelson condition in networks is to be made in the field of public finance. In particular, the worldwide trend of fiscal decentralization witnessed in the past few decades has given rise to an impressive and still growing empirically oriented literature, wherein the relationship between per capita income and fiscal decentralization is shown to be somehow dependent upon the stage of economic development in the country under analysis (see, for instance, Letelier, 2005, and Bodman and Hodge, 2010). It is of interest to utilize the extended Samuelson condition for local public goods to offer a new slant on the causality between economic development and fiscal decentralization. We shall elaborate on this point at the end of this paper, by invoking certain technical formulations to be introduced in Section 3.

Our analysis also demonstrates that independence of the optimal allocation from the distribution of resources at interior Samuelsonian allocations in networks generally fails to hold, even for the Bergstrom-Cornes preferences. The reason for this is that the production of the public good in the network is decentralized, undertaken at the individual level, and consequently the distribution of resources that is reflected by the individual agents' production set (or the PPF curves) translates into the Samuelsonian allocation. We shall turn to this point in much more detail in Section 5.3.

As is stated above, there is very limited extant analysis of cooperative solutions to the provision of public goods in the network. One remarkable exception is the recent study by Elliott and Golub (2015), wherein the authors' main interest is to explore the connection between the Pareto efficiency and centralities of a weighted and directed network that is constructed from the marginal benefits the individuals confer on one another. While this study contributes some important insights into the connection between the Walrasian prices and the players' centralities in the network defined as such when (positive) externalities are present, its focus and approach both differ substantially from that of the present study. Its main result, namely that the Lindahl solution to the public goods provision problem has a centrality property, which the authors obtain by invoking sophisticated analysis of network spectra and centralities, nonetheless closely relates to our result on the extended Lindahl equilibrium. More space is to be devoted to this point in Section 4 after our result on the Lindahl scheme in networks is formally established.

The rest of this paper is organized as follows. We first illustrate our main results by using some simple examples in the next section, and then lay down a model of the public good in networks and establish the generalized Samuelson condition in Section 3. Section 4 presents the extended Lindahl scheme for the provision of a public good in the network, showing the existence and optimality of the extended Lindahl equilibrium. Section 5 provides some further extensions and discussion, and Section 6 concludes.

## 2. The main results illustrated by simple examples

It appears useful to illustrate the main results of this paper on the Samuelson condition and the Lindahl scheme in networks by some simple examples, before a full-fledged treatment is offered in the next two sections.

[2] It is of interest to note that homogeneity promotes cooperation not only in human societies (see, e.g. Apicella et al., 2012) but also among social non-human animals such as chimpanzees (Massen and Koski, 2014).

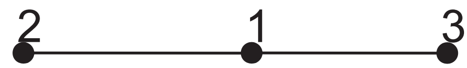


**Fig. 1.** A 3-person path network.

For the Samuelson condition in networks, let's consider a very simple 3-person community, wherein agent 1 is linked to both agents 2 and 3 and there is no link between the latter two (refer to Fig. 1). The adjacency matrix of the path network is therefore $A \equiv (a_{ij})_{3\times 3} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. Each individual is endowed with one unit of resource, which can be used to produce a private good $X$ or a local public good $G$, each with an identity production function. That is, for any individual $i$, $x^i + g^i \le 1$, where $x^i$ is her possible output of $X$ and $g^i$ that of $G$. Only she and her neighbor(s) benefit from her provision of $G$, and her preference is represented by a well-behaved utility function $u^i(x^i, G^i)$, where $G^i = \sum_j a_{ij} g^j$, the agent's consumption of the public good. The question of interest to us is how an interior Pareto solution $\{(x^i, g^i), i = 1, 2, 3\}$ looks. For that purpose, we may consider maximization of the utility of agent 1 without anybody else being worse off. Let $g \equiv [g^1, g^2, g^3]^T$ and $(Ag)_i$ be the $i-th$ component of vector $Ag$, $i \in \{1, 2, 3\}$. Then, the first-order condition of the Lagrangean function $\Sigma_{i=1}^{3}\lambda_i u^i(x^i, (Ag)_i)$, with $\lambda_i \equiv 1$, turns out to be

$$\begin{bmatrix} 1 - MRS^1_{X,G} & 1 & 1 \\ 1 & 1 - MRS^2_{X,G} & 0 \\ 1 & 0 & 1 - MRS^3_{X,G} \end{bmatrix} \begin{bmatrix} \lambda_1 u^1_G \\ \lambda_2 u^2_G \\ \lambda_3 u^3_G \end{bmatrix} = 0$$

where ${MRS_{X,G}}^i$ is individual $i$'s marginal rate of substitution of $X$ for $G$, $\forall i \in \{1, 2, 3\}$. Non-zero of each of ${\lambda_i u_G}^i$, $i \in \{1, 2, 3\}$ implies that the symmetric coefficient matrix of the above equation is singular, i.e.,

$$\begin{vmatrix} 1 - MRS^1_{X,G} & a_{12} & a_{13} \\ a_{21} & 1 - MRS^2_{X,G} & a_{23} \\ a_{31} & a_{32} & 1 - MRS^3_{X,G} \end{vmatrix} = 0$$

We shall show that this observation indeed holds more generally and reduces to the justly celebrated Samuelson condition when the public good is a global one (the provision by anyone benefits all.

The set of Pareto solutions $\{(x^i, g^i), i \in \{1, \ldots, n\}\}$ for a community made up of $n$ individuals is in general an infinite one. Among them, one of significance is what may be achieved by a Lindahl scheme, under which each agent faces a personalized market price (the Lindahl tax) of the public good. We extend such a scheme to public goods in network in a manner that may be seen as a natural one. For the sake of illustration, we may recycle the above example of a 3-person path network and assume a Cobb-Douglas utility function $u^i(x, G) = xG$ where $x$ is the consumption of a private good and $G$ is that of a local public good for each individual $i$. For any agent $i$, we solve the problem $\underset{G}{\text{Max}}\, u^i(1 - \tau G, G)$ for a given value of parameter $\tau$, representing the personalized price of the public good the agent faces, and denote by $\tau_i(G)$ the inverse demand function. A set $\{G^i, i = 1, 2, 3\}$ is mutually compatible if $\sum_j a_{ij}[\tau_j(G^j)G^j] = G^i$ holds for each $i$. A compatible set $\{G^i, i = 1, 2, 3\}$ is defined as a Lindahl equilibrium. At such equilibrium, $\tau_i(G^i)G^i$, $i \in \{1, 2, 3\}$ is agent $i$'s "purchase" (contribution) of the public good. It can then be easily shown that at Lindahl equilibrium defined as such, $G^1 = \frac{3}{2}$, $G^2 = G^3 = 1$, $\tau_1 = \frac{1}{3}$, $\tau_2 = \tau_3 = \frac{1}{2}$. That is, each allocates her resource between $X$ and $G$ half and half, but agent 1 is enabled to exploit her better position in the network, being better off compared to the rest of the community, since she faces a price of the public good (1/3) lower than that faced by her peers (1/2). It is to be noted that the personalized price for any agent $i$ at Lindahl equilibrium, $\tau_i$, equals ${MRS_{G,X}}^i = 1/{MRS_{X,G}}^i$, resulting in