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Numerical models and experimental investigation of energy loss mechanisms in SOI-based tuning-fork gyroscopes

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ABSTRACT

This paper presents numerical models and experimental investigation of energy loss mechanisms in tuning-fork gyroscopes fabricated on silicon-on-insulator (SOI) wafers. While the numerical model of thermoelastic damping is created according to a thermal-energy method, in which thermoelastic damping is interpreted from thermal perspective and its mathematical expression is derived using a well-defined thermodynamic parameter—entropy, the numerical model of anchor loss is based on a separation-and-transfer method, in which a tuning-fork structure and its substrate are first separated for analysis and then the stress from the clamped regions of the structure is transferred to the substrate for determining the vibration displacement across the clamped regions. The corresponding experimental investigation of energy loss mechanisms in the tuning-fork gyroscopes is consequently conducted. By comparing with the measured highest Quality factor (Q) values, the numerical models of thermoelastic damping and anchor loss are validated. The combination of the numerical models and the experimental measurement sheds important and interesting insight on the achievable Q value of these SOI-based tuning-fork gyroscopes: (1) from the design perspective, thermoelastic damping is the sole dominant loss in the gyroscope structure, given that the anchors are tightly fixed to the substrate; (2) but from the fabrication perspective, anchor loss can vary from negligible to significant, simply because HF etching causes the undercut underneath the anchors to various unpredictable extents. In addition, surface loss, damping from experimental electronics, and the effect of the DC polarization voltage on the measured Q values are also addressed.

1. Introduction

Micromachined gyroscopes are of great interest for a wide range of guidance, navigation, and control (GNC) applications [1–3]. The majority of micromachined gyroscopes are based on the Coriolis effect, where a rotation rate signal will cause vibration energy transferred from one resonant mode (drive-mode) to another (sense-mode) of a mechanical structure. Among various Coriolis-based micromachined gyroscopes developed so far, tuning-fork gyroscopes with integrated electrostatic transducers have been extremely attractive [4–8], because of their significant advantages, including large proof mass, large vibration amplitude in the drive-mode, and relative ease of fabrication, over other structural forms and other types of transducers. To achieve high-precision performance, energy loss mechanisms or quality factors (Q) of tuning-fork gyroscopes have been identified as the most critical design parameter, as a higher Q in such devices translates to improved rate resolution, higher rate sensitivity, and better bias stability [3,4,8,9]. Therefore, it is desirable to design and fabricate a tuning-fork gyroscope with high Q or little energy loss. Toward this end, one needs to understand energy loss mechanisms in such devices, not only for predicting their performance at the design stage, but also for improving their performance through design and fabrication trade-off.

Due to its small size, it is feasible to package a micromachined tuning-fork gyroscope in vacuum and thereby air damping is eliminated. Consequently, thermoelastic damping, anchor loss, and surface loss come to the fore and determine its Q value. For a tuning-fork gyroscope operating in vacuum, the measured Quality factor, Q_{measured}, of a tuning-fork gyroscope operating in vacuum is expressed as [4,10,11]:

$$\frac{1}{Q_{\text{measured}}} = \frac{1}{Q_{\text{unload}}} + \frac{1}{Q_{\text{electronics}}} = \frac{1}{Q_{\text{TED}}} + \frac{1}{Q_{\text{anchor}}} + \frac{1}{Q_{\text{surface}}} + \frac{1}{Q_{\text{electronics}}} \quad (1)$$

where Q_{TED}, Q_{anchor}, and Q_{surface} denote the Quality factor related to TED, anchor loss, and surface loss, respectively. Furthermore, Q_{unload} denotes the Quality factor resulting from the sum of these three loss mechanisms pertaining to the gyroscope itself. Note that $Q_{\text{electronics}}$

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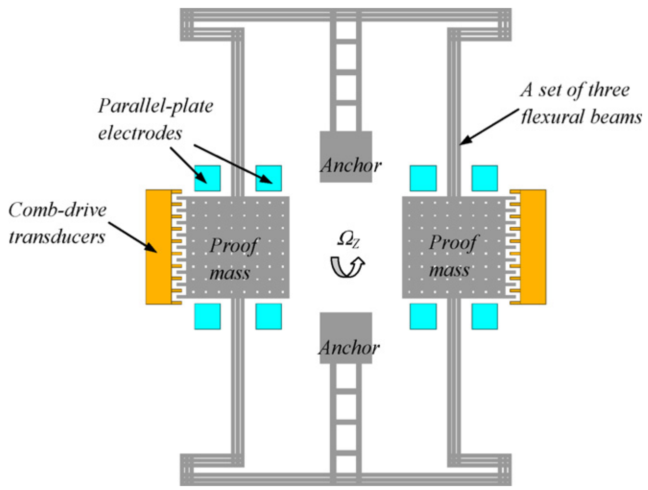

Fig. 1. Schematic view of a tuning-fork gyroscope design and operation.

is added to denote the damping related to experimental electronics, but this damping is not associated with the gyroscope design and fabrication.

Although these energy loss mechanisms are well known, analytical studies of these loss mechanisms have mostly targeted beam resonators [10–14], simply because of their simplicity in structure and theoretical analysis. In contrast, little progress has been made in calculating energy loss in tuning-fork gyroscopes [15,16]. Because thermoelastic damping and anchor loss are closely related to the structure design of a tuning-fork gyroscope, this paper focuses on developing the numerical models of these two loss mechanisms in tuning-fork gyroscopes fabricated on silicon-on-insulator (SOI) wafers. Since it is heavily dependent on the fabrication process used, surface loss will be addressed in conjunction with the experimental investigation.

This paper is organized as follows. Section 2 describes the design and operation of a tuning-fork gyroscope. Sections 3 and 4 present a numerical model of thermoelastic damping (TED) according to a thermal-energy method and a numerical model of anchor loss based on a separation-and-transfer method, respectively. The corresponding experimental investigation is detailed in Section 5. Then, in Section 6, the experimental measurement is compared with the numerical models, and important observations during experiment are discussed as well. Section 7 addresses surface loss, damping from experimental electronics, and the effect of the DC polarization voltage on the measured Q values. At the end, the significant insight drawn from the combination of the numerical models and the experimental investigation is concluded.

2. Design and operation

Fig. 1 shows a schematic view of a tuning-fork gyroscope design. This design consists of a tuning-fork structure and its integrated electrostatic transducers for excitation and detection. The tuning-fork structure comprises of a set of three flexural beams and two proof masses, and is fixed on the substrate through the two anchors located at the center of the whole device. Fig. 2 illustrates the resonant frequencies and vibration shapes of the two operation modes: drive-mode and sense-mode. While comb-drive transducers are employed in the drive-mode for achieving large drive-mode vibration amplitude, parallel-plate electrodes are incorporated in the sense-mode for detecting a rotation rate signal and minimizing the frequency difference between the drive-mode and the sense-mode [1,4,9].

To detect a rotation rate signal, first, the comb-drive transducers in the drive-mode are utilized to set up the drive-mode vibrations in the tuning-fork structure. Because of the Coriolis effect, a rotation rate signal excites the vibrations in the sense-mode and the vibration amplitude of the sense-mode is detected by the parallel-plate electrodes, leading to the detection of the rotation rate. To maximize the rate sensitivity by approximately a factor of Q in the sense-mode, the resonant frequencies of the drive-mode and the sense-mode are typically designed in proximity to each other. High Q values from both operation modes are desirable for the purpose of performance improvement.

3. Numerical model of thermoelastic damping

Thermoelastic damping in a mechanical structure is part of the vibration energy dissipated into thermal energy, through irreversible heat conduction accompanying elastic vibrations in the structure [17]. The governing equations of linear thermoelasticity have been well established [17]. Therefore, calculating thermoelastic damping is a well-defined problem—to solve the governing equations for the dissipation of vibration energy per cycle of vibration. To calculate thermoelastic damping, a well-accepted hypothesis [13,18,19,20,21,22,23] is that thermoelastic coupling is very weak and thus has negligible influence on the uncoupled elastic vibration modes of a mechanical structure. Hence, the elastic and thermal problems are essentially decoupled. By considering thermoelastic damping from the elastic wave perspective, a complex-frequency method, in which thermoelastic damping is formulated in terms of a complex-frequency value, has been developed for calculating thermoelastic damping [13,21,22,23]. However, this method incurs complex values in the calculation and leads to great complexity [20].

To avoid the complex values in calculating thermoelastic damping, we interpret thermoelastic damping from the thermal perspective and formulate it as the generation of thermal energy per cycle of vibration—referred as a thermal-energy method here. In fact, this method takes advantage of the very essence of thermoelastic damping: the dissipated vibration energy is permanently converted into thermal energy [20]. Consequently, thermoelastic damping is mathematically expressed with the aid of entropy—a well-defined thermodynamic parameter measuring generated thermal energy in irreversible heat conduction. In this section, we briefly review the governing equations of linear thermoelasticity associated with anisotropic materials, and describe the thermal-energy method and the numerical model of thermoelastic damping in a tuning-fork structure built upon this method.

3.1. Governing equation of linear thermoelasticity associated with anisotropic materials

For a mechanical structure that is initially at a uniform temperature, T_0, and is made from an anisotropic material, its three displacement components and temperature variation from the initial temperature are written as $u_i = (u_1, u_2, u_3)$, and $\theta = T - T_0$, respectively, in a Cartesian coordinate system $x_i = (x_1, x_2, x_3)$. Then, the governing equations of linear thermoelasticity for the mechanical structure are written as [24]:

$$\frac{1}{2} c_{ijkl}(u_{k,lj} + u_{l,kj}) - c_{ijkl} \cdot \alpha_{kl} \cdot \theta_{,j} = \rho \ddot{u}_i \quad (2a)$$

$$\kappa_{ij}\theta_{,jj} - \rho C_P \dot{\theta} = T_0 \beta_{ij} \dot{\varepsilon}_{ij} \quad (2b)$$

where $i, j, k, l = 1, 2, 3$; $\dot{\varepsilon}_{ij}$ is the rate of the corresponding elastic strain tensor at a point in the structure; c_{ijkl} is the fourth-order tensor of the elastic stiffness; α_{kl} is the thermal expansion tensor;