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Accurate modeling of air shear damping of a silicon lateral rotary micro-resonator for MEMS environmental monitoring applications

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ABSTRACT

The energy losses of a silicon lateral rotary micro-resonator operating in humid air were investigated with a three-dimensional finite volume model and compared to experiments as well as a simplified analytical model. The simulations can provide accurate modeling (within 5%) of air damping (the dominant energy loss source), unlike the analytical model capturing as little as 50% of the experimental values. The results showed that a significant portion of energy losses are associated with the complex three-dimensional path lines near the micro-resonator's edges that are not captured in the simplified analytical models. The simulations provide useful guidelines to design simple temperature and humidity sensors.

1. Introduction

Silicon (Si) micro-resonators are heavily used as chemical and biochemical sensors in gaseous and liquid environments [1–5]. Their principle of operation often relies on measuring small changes in resonance frequency (f_0) as a result of a mass change in a sensitive layer (for a given molecule) that has been added onto the Si structure. The sensitivity of these sensors can be affected by water vapor in uncontrolled environments [6,7]. For example, the quality factor, Q, an important parameter dictating the micro-resonator's sensing resolution, is affected by the temperature and humidity in atmospheric air [8]. Hence, the presence of a humidity sensor in the system may be required to properly sense other chemical species. Most microelectromechanical system (MEMS) humidity sensors are capacitive or resistive and rely on a polymer layer (such as polyimide) sensitive to water vapor [6,7,9–11]. The challenges associated with these types of MEMS humidity sensors include low hysteresis, fast response times, stability over time, and complex device structure, and are a major limitation to widespread miniaturized humidity sensors [10]. As such, simple Si micro-resonators may be efficient humidity sensor alternatives given their inherent sensitivity to temperature and humidity on f_0 and Q [8,12].

From the above considerations, proper characterization of the effects of temperature and humidity on Q is required to provide useful guidelines on the design of micro-resonating chemical or humidity sensors in atmospheric air. Many studies have focused on the damping characteristics of MEMS devices as a function of pressure [13–22], including the effect of temperature at low pressures [23–25]. However, very few have investigated the combined effects of temperature and humidity on the Q factor of MEMS resonators in atmospheric air. The present study focuses on the 3D finite volume modeling of air shear damping of a MEMS resonator whose dynamic behavior has already been characterized over a large range of humid environments [8,12]. Specifically, the Q of lateral rotary Si micro-resonators with a few geometrical variants was previously measured for temperatures ranging from 20 to 80 °C and relative humidity (RH) levels ranging from 20 to 90%, and compared to a simplified analytical model [8]. The analytical model could only capture 50–75% of the measured energy losses that mainly resulted from air shear damping [8]. Here we show that the 3D numerical model is much more accurate than the analytical model and can be used to accurately predict the environmental effects on Q. More importantly, the simulations provide critical insights regarding the sources of errors associated with the simplified analytical models that are often used to predict the MEMS' dynamic behaviors [5].

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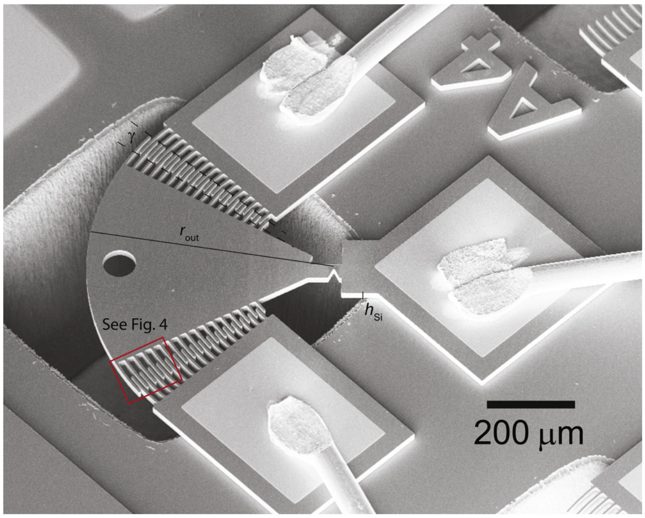

Fig. 1. Inclined SEM image of a micro-resonator.

2. Modeling

2.1. Description of the MEMS resonators

The MEMS structures (see Fig. 1) consist of lateral rotary micro-resonators, with a f_0 of either ~40 kHz or ~4.4 kHz, fabricated from a Si-on-insulator wafer [8]. The thickness of the Si layer, h_{Si}, is 10 or 25 μm. The structures are made of a notched cantilever beam and a fan-shaped mass (60° span, 30-μm inner radius, outer radius r_{out} = 300 μm (for the ~40 kHz resonators) or r_{out} = 900 μm (for the ~4.4 kHz resonators)) with two comb structures (16 (for r_{out} = 300 μm) or 66 (for r_{out} = 900 μm) interdigitated fingers, with a 3 μm gap (g = 3 μm) between the 3-μm-wide fingers, and a finger overlap, γ, of 2.5 or 5°) on each side; see Fig. 1. The mass has a 40-μm-diameter hole near its outer edge whose effect on Q is small and therefore neglected in this study [26]. These oscillators are through-hole structures, i.e. there is no substrate underneath the devices. Details regarding the testing of these micro-resonators and measurements of Q can be found elsewhere [8].

2.2. Analytical model

It was previously shown that the main source of energy dissipation of these lateral rotary micro-resonators is shear damping of the air: squeeze damping is negligible (the estimated squeeze number is negligible) and other sources (thermoelastic dissipation, anchor losses) as well (Q in vacuum is more than 40 times larger than in air) [8]. An analytical model was developed, based on the Navier–Stokes equation governing the air velocity profile, and the air was assumed to be an incompressible linear Newtonian fluid with dynamic viscosity μ [8]. The two main contributions of air shear damping are: (1) damping of the air below and above the plate (including moving fingers), $Q_{top/bot}$, and (2) damping of the air along the sidewalls of the moving fingers, Q_{comb}. The corresponding equations for tangential velocity (u_θ) and shear stress (τ) were derived as (see [8] for details):

$$u_\theta = r\omega\theta_0 e^{-z/\delta}\cos\left(\omega t - \frac{z}{\delta}\right) \quad (1)$$

$$\tau = \tau_{\theta z} = \mu\frac{\partial u_\theta}{\partial z} = -\frac{\sqrt{2}\mu.\omega.r.\theta_0}{\delta}e^{-z/\delta}\sin\left(\omega t - \frac{z}{\delta} + \frac{\pi}{4}\right) \quad (2)$$

to calculate $Q_{top/bot}$ (δ is penetration depth $\delta = \sqrt{\frac{2\nu}{\omega}}$, ω is the angular rate, θ_0 is the amplitude of rotation, r is the radial distance, z is the distance from the top surface of the device; see axes in Fig. 2).

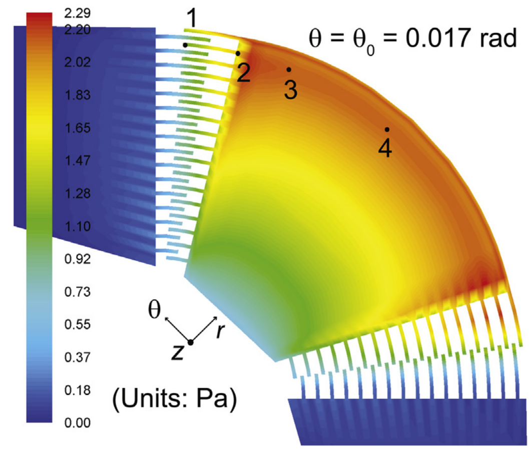

Fig. 2. Map of shear stress traction magnitude (in Pa) across the top surface of a micro-resonator (r_{out} = 300 μm, γ = 2.5°, h_{Si} = 25 μm) for $\theta = \theta_0$ = 0.017 rad.

Table 1
List of J and ω values used to evaluate Q with the models.

Geometry	J (kg m^2)	ω (rad s^{-1})
r_{out} = 300 μm; h_{Si} = 10 μm	5.25×10^{-17}	252,980
r_{out} = 300 μm; h_{Si} = 25 μm	1.30×10^{-16}	258,635
r_{out} = 900 μm; h_{Si} = 10 μm	4.30×10^{-15}	27,910

To calculate Q_{comb}, a linear distribution of the tangential velocity between a fixed finger and an adjacent moving finger was assumed, leading to the following equation [8]:

$$u_\theta = \frac{r_1(r + g - r_1)\omega\theta_0\cos(\omega t)}{g} \quad (3)$$

where r_1 is the distance between the point at which u_θ is calculated and the nearest adjacent fixed finger, and the corresponding shear stress was in the form:

$$\tau = \tau_{\theta r} = \mu\frac{r\omega}{g}\theta_0\cos(\omega t) \quad (4)$$

Once u_θ and τ were calculated, Q was calculated using:

$$Q = 2\pi\frac{E_{stored}}{E_{loss}} \quad (5)$$

with

$$E_{stored} = \frac{1}{2}J(\omega\Theta_0)^2 \quad (6)$$

where J (the mass moment of inertia) and ω values for each geometrical variants are given in Table 1 (see [8]). E_{loss} was calculated using:

$$E_{loss} = \frac{1}{\omega}\int_0^{2\pi}\int \tau u_\theta dS\, d(\omega t) \quad (7)$$

where τ is the shear stress corresponding to the considered source of shear damping and u_θ is the velocity of the air, both terms taken at the micro-resonator's surface (with elementary area dS) corresponding to the considered source of damping. It is important to emphasize that, in this analytical model, it was assumed that the micro-resonator was an infinitely large plate, resulting in the tangential component of velocity (using cylindrical coordinates) being the only non-zero component. As will be shown with the numerical model, this assumption leads to a significant under-estimation of the energy losses.