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Sustainability with endogenous discounting when utility depends on consumption and amenities

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HIGHLIGHTS

- For constant utility, discounting must be endogenous.
- The discount rate is adjusted for consumption growth and substitution.
- Hartwick's rule remains valid.
- Hotelling's rule is adjusted for amenity value.

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ABSTRACT

We study the issue of sustainability using a model with a stock of man-made capital and a stock of exhaustible natural resource that provides a flow of amenity services as well as an input for the production of a consumption good. We ask under what conditions the utility flow will be a constant if infinitesimal households discount their utility using an endogenous utility discount rate that depends some macroeconomic variables. Our main result is that for the utility flow to be constant, the utility discount function must be the marginal product of capital function adjusted for the growth rate of aggregate consumption weighted by the elasticity of the consumer's marginal rate of substitution between the final good and the amenity services. We demonstrate that Hartwick's Rule holds but the Hotelling Rule must be modified. We also provide an explicit analytical example to confirm the general result.

1. Introduction

The rapid depletion of environmental assets has led social scientists to develop criteria for sustainable development (Solow, 1974; Hartwick, 1977; Pezzey, 1992; Martinet, 2012; Cairns and Martinet, 2014; Asheim, 2010; Mitra et al., 2013; Fleurbaey, 2015; Figuières et al., 2017; Long and Martinet, 2018). An appealing sustainability concept is that of maintaining a constant flow of utility.[1] There is however a stumbling block: the standard model of infinitely lived individuals assumes that the utility discount rate is a constant, and this is generically inconsistent with maintaining a constant utility flow over the whole program. In fact, Dixit et al. (1980) show that along any path of resource extraction and capital accumulation that ensures constant consumption, the consumer's implicit rate of utility discount must vary over time. However, they do not say where such a time-varying utility discount rate might come from. This raises an interesting question: Is it possible to trace this non-constancy to some formula that links the utility discount rate to some underlying fundamentals? For example, the utility discount rate at time t may depend on the rate of growth of per capita consumption, or the quality of the environment at that time.[2]

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[1] For simplicity, in this paper, a sustainable path is taken to mean an efficient path that ensures a constant stream of utility. This is in line with who defined sustainability as referring to " an obligation to conduct ourselves so that we leave to the future the option or the capacity to be as well off as we are." A somewhat different definition would be to take sustainability as meaning a non-decreasing stream of utilities (see. e.g., Asheim, 2007; Cairns and Martinet, 2014). For an insightful review of various concepts of sustainability, and a novel definition in terms of sustaining certain defined targets, see Fleurbaey (2015). In the concluding section, we will make some comments relating our result to these differing concepts of sustainability.

[2] There is related literature where authors postulate that the utility discount rate depends on some endogenous variable (e.g. Uzawa, 1968; Boyer, 1975; Epstein, 1987; Obstfeld, 1990; Pittel, 2002, Ch. 5; Ayong le Kama and Schubert, 2007; Yanase, 2011).

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In this paper, we consider an economy populated with infinitesimal households that have a utility function that depends on a produced consumption good as well as a flow of services called environmental amenities, as in Krautkraemer (1985), and d'Autume and Schubert (2008). We posit that consumers use a utility discount function that may depend on some macroeconomic variables, but we do not assume any specific form of dependence. We demonstrate that for constant utility to hold, the utility discount function must be such that the endogenous utility discount rate equals the marginal product of the aggregate capital stock adjusted for the growth rate of aggregate consumption weighted by the elasticity of the marginal rate of substitution between the final good and the amenity services with respect to the consumption. We show that under these circumstances, Hartwick's Rule (Hartwick, 1977) holds, while the Hotelling Rule (see, e.g., Gaudet, 2007) must be modified. We also provide an explicit analytical example to illustrate our general result.

Our paper is related to a stream of literature, which perhaps may be described as "reverse engineering", that asks the following type of questions: given an observed (or perhaps hypothesized) set of facts (or properties), can we work backward to discover some hidden properties of the underlying preference structure that would account for it? One of most striking examples of this literature is Koopmans (1960), who shows that if an individual is able to rank all possible consumption paths with a stationary utility function satisfying continuity, sensitivity, absence of intertemporal complimentarity, and existence of a best and a worst program, then her preference must involve some form of impatience.[3]

Related to the reverse engineering literature is the work of Arrow et al. (1961). They asked, what does the production function look like, if it generates factor demand functions that display a constant elasticity of substitution? By integrating, they discovered the form of the CES function. On a similar vein, researchers in public-sector economics often ask: given the observed policy decisions (such as a country's tax structure), can one work backward to identify the objective function of the government? This type of research sometimes involves inverting formulas from the Mirrlees optimal income tax framework, such as proposed in Diamond (1998) and Saez (2001), and implemented by Bourguignon and Spadaro (2012), Bargain et al. (2014) and Lockwood and Weinzierl (2015).

The plan of the paper is as follows. Section 2 introduces the model and derives the main results. Section 3 provides an explicit analytical example. Section 4 offers some concluding remarks.

2. The model

Following Krutilla (1967), Krutilla and Fisher (1975), Krautkraemer (1985) and d'Autume and Schubert (2008), we consider the case where the stock of natural capital has a dual function: the flow of extraction serves as input to the aggregate production function and while the stock provides amenity services for consumers, who enjoy the recreational and aesthetic value of preserved environments. As in Krautkraemer (1985) and d'Autume and Schubert (2008), we treat the stock of resource as non-renewable. Old growth forests, for example, are practically an irreplaceable natural capital, and may be classified as an exhaustible resource. Similarly, sand-stone cliffs, once quarried to make buildings, cannot be restored.

We consider a continuum of infinitely lived individuals, indexed by θ, where $\theta \in [0, 1]$. Each individual θ is endowed with an initial capital stock $k(0, \theta)$ and a (privately owned) exhaustible resource stock, $x(0, \theta)$. At any time t, each individual derives utility from her consumption of the final good, $c(t, \theta)$, and from the amenity services provided by their own resource stock. By choice of units, the service flow is also denoted by $x(t, \theta)$. Individuals' utility function is denoted by $u(c(t, \theta), x(t, \theta))$ where t denotes time. We assume that $u(.)$ is strictly increasing and strictly concave, with $u_{cx} > 0$, $u_c(0, x) = \infty$ and $u_x(c, 0) = \infty$.

Let $q(t, \theta)$ be the individual's extraction from stock $x(t, \theta)$. The dynamics of the resource stock is

$$\dot{x}(t, \theta) = -q(t, \theta)$$

The individual sells the extracted resource at the market price $p(t)$. Firms buy the extracted resource and use it as input in the production of the final good, which can be consumed or invested. Firms do not own capital: they rent capital from individuals, at the market rental rate $r(t)$. The economy's aggregate capital stock is $K(t)$, where

$$K(t) \equiv \int_0^1 k(t, \theta) d\theta$$

Define the aggregate resource input by

$$Q(t) \equiv \int_0^1 q(t, \theta) d\theta$$

The aggregate production function is

$$Y(t) = F(K(t), Q(t))$$

where $Y(t)$ is the output of the final good.[4] The production function has the usual properties: concavity, positive and diminishing marginal products, and the Inada conditions hold. We also assume that $F(.)$ is homogeneous of degree 1, so that firms earn zero profit. Perfect competition prevails, so that

$$F_K(K(t), Q(t)) = r(t) \tag{1}$$

$$F_Q(K(t), Q(t)) = p(t) \tag{2}$$

The individual's income at time t consists of the revenue from the sales of the extracted resource and the rental income from the capital stock:

$$y(t, \theta) = r(t)k(t, \theta) + p(t)q(t, \theta)$$

Individuals' capital accumulation equation is given by

$$\dot{k}(t, \theta) = y(t, \theta) - c(t, \theta)$$

The economy's aggregate resource stock is

$$X(t) \equiv \int_0^1 x(t, \theta) d\theta.$$

The aggregate consumption of the produced final good is

$$C(t) = F(K(t), Q(t)) - \dot{K}(t)$$

and its growth rate is denoted by $g(t)$

$$g(t) \equiv \frac{\dot{C}(t)}{C(t)}$$

3 In a related paper, assuming non-additive utility, Boyer (1975) shows that the implicit utility discount rate in general varies along a given growth path. Since the implicit utility discount rate varies with the capital per capita, the steady state is not unique and the long-run state of the economy depends on its point of departure. However, the optimal sequences for per capita consumption and capital are unique and converge monotonically. The author uses a dynamic programming formulation which permits the reduction of the infinite-horizon problem to a sequence of two-period problems.

4 Solow (1974) and Dasgupta and Heal (1979) assume that the production function is Cobb–Douglas.