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von Neumann–Morgenstern stable sets of a patent licensing game: The existence proof[☆]

Toshiyuki Hirai [a], Naoki Watanabe [b,*]

[a] Faculty of Economics, University of Toyama, 3190 Gofuku, Toyama 930-8555, Japan
[b] Graduate School of Business Administration, Keio University, 4-1-1 Hiyoshi Kohoku, Yokohama, Kanagawa 223-8526, Japan

HIGHLIGHTS

- In our patent licensing game, a license negotiation is formulated as a game with a coalition structure.
- We provide the sufficient conditions for the existence of stable sets in such a game where the core may be empty.
- The key feature is to consider whether stable sets of reduced games can construct stable sets of the original entire games.

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ABSTRACT

This paper provides the existence proof for stable sets of a game which may have empty cores. Given the number of licensees of a patented technology which is determined by the patent holder without any production facilities, a game with a coalition structure is formulated with the outcome expected in the subsequent market competition where any cartels are prohibited. Although the core is non-empty if and only if the grand coalition is formed with a condition, we provide, for each permissible coalition structure, the sufficient condition(s) for the existence of von Neumann–Morgenstern stable sets of the game. Under symmetric imputations, there exist stable sets for any permissible coalition structures, and each of those is completely characterized.

1. Introduction

This paper considers bargaining outcomes in prices of information on patented technologies under a situation where the seller of the information (patent holder) has no production facility and the information causes the non-buyers (non-licensees) a negative externality through the market competition with the buyers (licensees). Those licensing agreements are, basically, contract terms signed by sellers and buyers of information that result from negotiations. We thus investigate the existence and some properties of von Neumann–Morgenstern stable sets of a patent licensing game which was developed by Watanabe and Muto (2008) as a game with a coalition structure.[1]

von Neumann and Morgenstern (1944) considered social systems stylized in real practices as outcomes of negotiations made by people who are faced with non-cooperative situations, and proposed a solution which describes agreements people reach eventually in those negotiations, which can be interpreted as an accepted standard of behavior. The solution is what we currently call a von Neumann–Morgenstern stable set, or simply a stable set. It is, however, well known that "finding stable sets involves a new tour de force of mathematical reasoning for each game or class of games that is considered" (Aumann, 1987, p. 59). In fact, the general existence condition of stable sets has not been found yet, since Lucas (1968, 1969) gave an example of a ten-player game with no stable set.

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* Corresponding author.
E-mail addresses: thirai@eco.u-toyama.ac.jp (T. Hirai), naoki50@keio.jp (N. Watanabe).

[1] Patent licensing had been investigated with non-cooperative licensing mechanisms. See Sen and Tauman (2007), Fan et al. (2016), and the references therein. Kishimoto (2013) extended the model in Watanabe and Muto (2008) by using a game without side-payment.

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For this difficulty, the door was recently pushed open a crack in economic applications of non-symmetric games.[2] For example, Wako (2010) proved that every marriage game has a unique stable set, referring to its property shown by Ehlers (2007).[3] Núñez and Rafels (2013) showed the existence of stable sets in assignment games, searching for the way patiently after a result proven by Solymosi and Raghavan (2001).[4] In essence, many of those researchers constructed other games for an auxiliary purpose in such a way that the cores in those games coincide with the ones in the original games and then showed that the cores in the auxiliary games are themselves the unique stable sets in those games corresponding to the original games.[5] This strand of research is, in a broad sense, called the "core stability" problem.[6]

The purpose of this paper is to take a small step forward in the existence proof of the stable sets of games with empty cores. We analyze what range of payoffs the patent holder can guarantee to him or herself as stable bargaining outcomes when it determines the number of licensees and proposes his or her own payoff. Given the number of firms which is determined by the patent holder to start with license negotiations, a game with a coalition structure is formulated with the outcome expected in the subsequent competition in the market as a game with a coalition structure. Our results in this model are as follows. The cores are non-empty if and only if the grand coalition is formed under a condition. We provide, for each permissible coalition structure, the sufficient condition(s) for the existence of stable sets of the game. Under symmetric imputations, stable sets exist for any permissible coalition structures, and each of those is completely characterized.

The key feature of our existence proof is to consider whether stable sets of "reduced games" can construct stable sets of the original games in which the core is empty. Thus, as the first step, we search for some condition(s) with which cores of those reduced games are non-empty and stable. In this sense, our step taken in this paper is based on the heritage of the past research on core stability mentioned above, though few papers in the literature considered to construct a stable set from stable sets of reduced games.[7] In our model, moreover, the core may not coincide with the stable set in the original game, even if it is non-empty.

The remaining part of this paper is organized as follows. Section 2 describes a patent licensing game, formulates a license negotiation from the game, and defines solution concepts for the negotiation. Section 3 analyzes the license negotiation and provides major results. At the end of this section, we provide a case in our model where the core is non-empty but it is strictly included in the stable set. Section 4 restricts our attention to symmetric payoffs for licensees. Section 5 closes this paper with some remarks.

2. Model

2.1. Patent licensing

Let $N = \{1, 2, \ldots, n\}$, where $2 \leq n < \infty$, be the set of firms that have an identical production technology before a patented technology is licensed.[8] There is an agent who holds a patent of a new technology for which those firms have a demand. This agent does not have appropriate production facilities; thus it cannot receive revenue from the patented technology unless it sells the licenses to firms. This agent is called an external patent holder.[9] The set of players of this game is $\{0\} \cup N$, where the external patent holder is denoted by player 0. Assume that the patent is perfectly protected; no firm can use the patented technology without the patent holder's permission.

The patent licensing is here modeled as a situation with three stages. At stage (i), the patent holder selects a subset $S \subset N$ and invites the firms in S to negotiate on license issues. We assumed above that firms have an identical production technology before a patented technology is licensed. Thus, selecting a subset S of firms is choosing the number (integer) of licensees. Firms in $N \setminus S$ cannot participate in the negotiations. At stage (ii), every firm in S negotiates with the patent holder over how much it should pay as the fee to the patent holder. The patent holder and firms in S may communicate among themselves in their negotiations, but firms in $N \setminus S$ cannot observe how the negotiations run. When some firms in S fail to reach agreements on the fees in their negotiations, all the negotiations among players in $\{0\} \cup S$ break off. The patent holder can then license his or her patented technology to any other firms at any rates of fee; otherwise, firms in $N \setminus S$ are not licensed. The payment to the patent holder is made at the end of this stage.[10] At stage (iii), firms compete in the market, knowing which firms are licensed or not. Licensees use the patented technology, while non-licensees use the old technology. Firms are prohibited from forming any cartels to coordinate their production levels and market behaviors, as is assumed in the traditional literature on patent licensing.[11]

The model stated above is analyzed backwardly from stage (iii) to stage (i). In the traditional models of patent licensing, take-it-or-leave-it offers are made from the patent holder to some or all firms

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2 As for the existence of stable sets in symmetric games, Shapley (1959) considered the stable sets of glove-market games. Hart (1973) provided a sufficient condition for the existence of the symmetric stable set in a typical production economy. Muto (1982) considered the symmetric stable set in an extension of that production economy called an (n, k)-game. Shapley (1973) provided a necessary and sufficient condition for the core to be the unique stable set in a symmetric game in his unpublished manuscript. Biswas et al. (2000) gave another proof to that result. The original proof by Shapley can be read in Suzuki and Muto (1985), although it is translated in Japanese.

3 Ehlers (2007) showed that if there exists a stable set in a marriage game, the set is a maximal distributive lattice of matchings that includes all core matchings. Herings et al. (2017) recently considered the stable sets with particular properties in marriage games.

4 Solymosi and Raghavan (2001) proved that the core of an assignment game is the stable set if and only if the assignment matrix has a dominant diagonal. Starting with this result, Núñez and Rafels took two elaborate steps (Núñez and Rafels, 2002, 2009) before showing the existence of stable sets in assignment games in Núñez and Rafels (2013), where they showed that the stable set is the union of the cores for subgames related to the original game. Bedney (2014) characterized the stable set in a one-seller assignment game as a graph of a continuous and monotone function.

5 In other applications, Champsaur (1975) showed the core stability in a public good economy. Hirai (2008) characterized the stable set in a public good economy where each coalition is allowed to achieve an allocation via a proportional income tax. Shitovitz and Weber (1997) considered the relationship between the graph of an equal-treatment Lindahl mapping and the stable set in a continuum economy. Einy and Shitovitz (2003) showed that, under some assumptions, the set of symmetric efficient allocations in a finite exchange economy is a unique symmetric stable set.

6 In the general class of TU games, Kikuta and Shapley (1986) introduced a sufficient condition for core stability, which is called extendability; a game is extendable if, for every subgame, each core element of the subgame can be extended to a core element of the game. Shellshear and Sudhölter (2009) relaxed a requirement of subgames for extendability. Azrieli and Lehrer (2007) showed that core largeness is equivalent to a stronger version of the extendability. Other papers related to the largeness of the core are Muto (1983), van Gellkom et al. (1999), and Biswas et al. (2001). As for the core stability in games with an infinite set of players or in non-atomic games, see, e.g., Einy and Shitovitz (1996) and Einy et al. (1996), which are based on the analysis in Hart (1974).

7 Peleg (1986) showed the core stability of an ordinary convex NTU game by using the core stability of a reduced game. Our approach is different from this in the sense that we construct a stable set in a TU game with an empty core by using the core stability of a reduced game.

8 We keep this assumption in the traditional literature of patent licensing intact.

9 Research laboratories and engineering departments at universities are typical examples of such agents, because they do not have any production facilities.

10 No negotiation process is specified at stage (ii), but the patent holder might negotiate with each firm in S on a one-by-one basis repeatedly. See, e.g., chapter 10 in Peleg and Sudhölter (2007).

11 See Tauman and Watanabe (2007) for an analysis in the case of firms being allowed to form cartels in a linear Cournot market.