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When do utilitarianism and egalitarianism agree on evaluation? An intersection approach

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HIGHLIGHTS

- We examine the intersection of utilitarian and leximin orderings.
- We characterize it using a new axiom on a composite utility transfer.
- We also examine the lexical compositions of utilitarian and leximin orderings.
- The lexical compositions are jointly characterized with additional axioms.

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ABSTRACT

We examine the range of the agreement between the utilitarian social welfare ordering (SWO) and leximin SWO by analyzing the intersection of them. We characterize the intersection (in terms of subrelation) using the strong version of Pigou–Dalton equity and a new axiom on the composition of rank-preserving progressive and regressive utility transfers. Then, adding separability and cardinal full comparability, we jointly characterize the leximin SWO and the lexicographic composition of the utilitarian and leximin SWOs that applies the utilitarian SWO first. We also jointly characterize these two SWOs and the utilitarian SWO using the weak version of Pigou–Dalton equity.

1. Introduction

A social welfare ordering (SWO) for vectors of individuals' utilities is a useful concept for evaluating the distributions of individuals' utilities or the social alternatives underlying those utility distributions. Of the many SWOs, two rooted in moral philosophy have drawn great attention, namely, the utilitarian SWO and the leximin SWO. The utilitarian SWO formalizes classical utilitarianism advocated by Bentham (1789) and evaluates utility vectors by comparing the utility sums. The leximin SWO is an egalitarian principle and a lexicographic modification of the maximin principle proposed by Rawls (1971).

The properties distinguishing the utilitarian and leximin SWOs have been studied using the axiomatic approach.[1] These SWOs are known to contrast each other in three respects: (i) measurability and interpersonal comparability of utilities; (ii) an equity property when two individuals' interests are in conflict; and (iii) a non-interference property when a change of utility vectors affects only one individual.[2] In respect of (i), while the utilitarian SWO is compatible with cardinally measurable and interpersonally unit-comparable utilities, the leximin SWO is compatible with ordinally measurable and interpersonally full-comparable utilities (d'Aspremont and Gevers, 1977). With regard to (ii), the utilitarian SWO is neutral as long as the utility differences are the same for conflicting individuals (Blackorby et al., 2002). Meanwhile, the leximin SWO prefers the utility inequality between conflicting individuals to diminish as long as their relative ranking is preserved (Hammond, 1976, 1979). With regard to (iii), Mariotti and Veneziani (2009, 2013) show that while the utilitarian SWO respects the affected individual's preference as long as her/his utilities change by the same amount in a pair of utility vectors, the leximin SWO does so if the individual's utilities worsen.

On the other hand, the properties common to utilitarian and leximin SWOs are given by five axioms in Deschamps and Gevers (1978). According to Deschamps and Gevers, if an SWO satisfies

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[1] Reviews of the literature on axiomatic analyses of SWOs are presented by Blackorby et al. (2005), Bossert and Weymark (2004), and d'Aspremont and Gevers (2002).

[2] In addition to these, Sen (1974) discusses the contrast between utilitarianism and the Rawlsian maximin and leximin principles in the context of dividing a fixed amount of income among individuals. Further, Bossert and Suzumura (2016) contrast utilitarianism with the Rawlsian maximin principle from the viewpoint of their relevance to the plurality voting rule.

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strong Pareto, anonymity, minimal equity, separability, and cardinal full comparability, then the SWO is either the leximin SWO or a weakly utilitarian SWO, that is, an SWO that respects the strict preferences of the utilitarian SWO.[3] Strong Pareto and anonymity postulate positively sensitive and impartial evaluations, respectively. Minimal equity formalizes a very weak equity property when two individuals' interests are in conflict. Separability requires the evaluation to be independent of utility-unconcerned individuals. Finally, cardinal full comparability corresponds to the assumption of cardinally measurable and interpersonally full-comparable utilities.

This paper follows Deschamps and Gevers (1978), but from a different point of view. We examine the agreement of the evaluations of utilitarian and leximin SWOs. To this end, we follow the *intersection approach* proposed by Sen (1973). In the context of inequality measurement, Sen argues as follows:

> ... it is significant to note that the alternative indicators tend to involve some conflicts and some corroboration of each other. We can sort out the picture of partial correspondence by taking the intersection of the set of chosen measures. (Sen, 1973 p. 72)

This approach can be applied to the analysis of SWOs.[4] Hence, we consider the intersection of utilitarian and leximin SWOs. The intersection has been analyzed by Blackorby and Donaldson (1977). The intersection of utilitarian and leximin SWOs is an intermediate egalitarian social welfare quasi-ordering (SWQ) between the generalized Lorenz SWQ proposed by Shorrocks (1983) and the leximin SWO.[5] Specifically, it is an extension of the generalized Lorenz SWQ (Blackorby and Donaldson, 1977) and is a subrelation of the leximin SWO.[6] However, to the best of our knowledge, an axiomatic foundation of the intersection of utilitarian and leximin SWOs is unknown. The principal purpose of the paper is to axiomatize it: in other words, we explore an axiomatic basis for the agreement between utilitarian and leximin SWOs.

To axiomatize the intersection of utilitarian and leximin SWOs, we introduce a new equity axiom that we call the composite transfer principle. Similar to the transfer sensitivity axiom in Shorrocks and Foster (1987), the composite transfer principle refers to the composition of rank-preserving progressive and regressive transfers involving three individuals. It asserts that the composition of a rank-preserving progressive transfer from the second worst off of the three to the worst off and a rank-preserving regressive transfer from the second worst off to the better off weakly increases social goodness. We show that the class of SWQs that are monotone with respect to the evaluations of utilitarian and leximin SWOs, which we call $R_{U,L}$-monotone SWQs, is characterized by strong Pareto, anonymity, the weak version of Pigou–Dalton equity, and the composite transfer principle. Further, we characterize the intersection of utilitarian and leximin SWOs (in terms of subrelation) by strengthening the weak version of Pigou–Dalton equity to its strong version.

Following Deschamps and Gevers (1978), we also examine $R_{U,L}$-monotone SWOs and ordering extensions of the intersection of utilitarian and leximin SWOs that satisfy separability and cardinal full comparability. We show that an $R_{U,L}$-monotone SWO satisfies separability and cardinal full comparability if and only if it is any one of the following three: the utilitarian SWO, the leximin SWO, and the lexicographic composition of utilitarian and leximin SWOs that applies the utilitarian SWO first. Further, we show that an ordering extension of the intersection of utilitarian and leximin SWOs satisfies separability and cardinal full comparability if and only if it is either the leximin SWO or the lexicographic composition of utilitarian and leximin SWOs that applies the utilitarian SWO first. Since the leximin SWO is the lexicographic composition of the utilitarian and leximin SWOs that applies the leximin SWO first, this result means that SWOs that both satisfy the common axioms of the utilitarian and leximin SWOs in Deschamps and Gevers (1978) and respect the agreement of the two SWOs are limited to their lexicographic compositions.

The rest of the paper is organized as follows. In Section 2, we present notation and basic definitions. Section 3 characterizes an $R_{U,L}$-monotone SWQ and the intersection of utilitarian and leximin SWOs. The joint characterization results of the utilitarian SWO and the lexicographic compositions of utilitarian and leximin SWOs are also presented. In Section 4, we discuss an alternative representation of the agreement between the utilitarian and leximin SWOs. We also show that it is impossible to obtain a continuous or representable ordering extension of the intersection of the utilitarian and leximin SWOs. Section 5 concludes the study. All proofs are relegated to Appendix A. The independence of the axioms in our axiomatizations is proved in Appendix B.

2. Basic framework

2.1. Preliminaries

Let $\mathbb{R}$ be the set of all real numbers and $\mathbb{R}_{++}$ be the set of all positive real numbers. The sets of all integers and all positive integers are denoted by $\mathbb{Z}$ and $\mathbb{Z}_{++}$, respectively. Given sets A and B of objects, we write $A \subseteq B$ to mean that A is a subset of B and $A \subset B$ to mean that $A \subseteq B$ and $A \neq B$.

Let $N = \{1, \dots, n\}$ be the set of n individuals, where $n \geq 2$. The set of all possible utility vectors $x = (x_1, \dots, x_n)$ for N is the n-dimensional Euclidean space $\mathbb{R}^n$, where x_i is the utility level of individual i. For all $x \in \mathbb{R}^n$, $(x_{(1)}, \dots, x_{(n)})$ denotes a rearrangement of x such that $x_{(1)} \leq \cdots \leq x_{(n)}$, with the ties being broken arbitrarily. For all $i \in N$, let e^i denote the ith unit vector in $\mathbb{R}^n$, that is, 1 in the ith place and 0 elsewhere. Our notation for vector inequality is as follows: for all $x, y \in \mathbb{R}^n$, $x \geq y$ if $x_i \geq y_i$ for all $i \in N$; $x > y$ if $x \geq y$ and $x \neq y$.

A binary relation R on $\mathbb{R}^n$ is a subset of $\mathbb{R}^n \times \mathbb{R}^n$. For convenience, we write xRy to mean $(x, y) \in R$. Given a binary relation R, its asymmetric and symmetric parts are denoted by P and I, respectively, that is, xPy if and only if xRy and not yRx, and xIy if and only if xRy and yRx. An SWQ on $\mathbb{R}^n$ is a reflexive and transitive binary relation on $\mathbb{R}^n$. An SWO on $\mathbb{R}^n$ is a complete SWQ on $\mathbb{R}^n$. Given SWQs R_1 and R_2 on $\mathbb{R}^n$, we say that R_1 is a *subrelation* of R_2 if for all $x, y \in \mathbb{R}^n$, (i) xI_1y implies xI_2y and (ii) xP_1y implies xP_2y. Conversely, we say that R_1 is an *extension* of R_2 if R_2 is a subrelation of R_1. Further, R_1 is said to be an *ordering extension* of R_2 if R_1 is an SWO and an extension of R_2.

2.2. The intersection of utilitarian and leximin SWOs

We first define the utilitarian and leximin SWOs. The utilitarian SWO on $\mathbb{R}^n$ is defined as the following binary relation R_U: for all $x, y \in \mathbb{R}^n$,

$$xR_Uy \Leftrightarrow \sum_{i \in N} x_i \geq \sum_{i \in N} y_i. \tag{1}$$

If an SWO R satisfies $P_U \subseteq P$, we say that R is weakly utilitarian.

[3] Related results are obtained by Ebert (1988), Gevers (1979), Roberts (1980), and Segal and Sobel (2002).

[4] Sen (1985) uses this approach to evaluate an individual's functionings. Further, Blackorby et al. (1996b) examine the intersection of orderings for variable-dimensional utility vectors. On this, see also Trannoy and Weymark (2009).

[5] A quasi-ordering is a reflexive and transitive binary relation.

[6] The definitions of extension and subrelation are given in Section 2.