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Economic growth and factor substitution with elastic labor supply

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HIGHLIGHTS

- I examine the effect of factor substitution on steady-state income and capital.
- Positive link if the baseline effective capital is below its steady state.
- Steady-state labor supply is decreasing in the elasticity of substitution.
- Steady-state capital income share is increasing in the elasticity of substitution.

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ABSTRACT

We study the link between the elasticity of factor substitution and economic growth in the Ramsey–Cass–Koopmans model with elastic labor supply and normalized CES production. If the baseline value of capital per unit of effective labor is below its steady-state value, an increase in the elasticity of substitution generates a higher steady-state income, capital and consumption per capita. This is due to the combination of a positive efficiency effect of a higher elasticity of substitution and a positive distribution effect. However, the effect of a higher elasticity of substitution on these variables along the transition is ambiguous.

1. Introduction

Is the elasticity of substitution an engine of growth? de La Grandville (1989) and Klump and de La Grandville (2000) show that the answer is affirmative in the Solow (1956) model: a higher elasticity of substitution generates a higher per capita capital and per capita income both in the steady state and along the transition. Klump (2001) proves that if the baseline per capita capital is below its steady-state value –the most plausible case – a higher elasticity of substitution also causes a higher steady-state per capita capital and a higher per capita income in the Ramsey–Cass–Koopmans (RCK) model. Miyagiwa and Papageorgiou (2003) find that such positive relationship does not necessarily hold in the Diamond (1965) overlapping-generations model. Irmen and Klump (2009) reconcile these findings by introducing possible asymmetries of savings out of factor income. They show that factor substitution has a positive efficiency effect and an ambiguous distribution effect on output. If the savings rate out of capital income is sufficiently high the efficiency effect dominates and the overall effect is positive. Xue and Yip (2012) present a comprehensive characterization of the link between the elasticity of substitution and the steady-state capital and output per capita in the Solow, Ramsey–Cass–Koopmans and Diamond models.

These works have clarified the nexus between factor substitution and economic growth in several 'classic' growth models. However, they have relied on growth models in which labor supply is inelastic. The purpose of this paper is to overcome this restrictive assumption by considering a framework –the Ramsey–Cass–Koopmans (RCK) model – in which labor supply is endogenous. We find that per capita (and aggregate) steady-state income, capital and consumption are increasing in the elasticity of substitution if the baseline effective capital (the ratio of per capita capital to labor supply) is below its steady-state value, which is the most plausible case. This is due to the combination of a positive efficiency effect (Klump and de La Grandville, 2000) – i.e., a higher elasticity of factor substitution increases the productivity of inputs – and a positive distribution effect (Irmen and Klump, 2009). The distribution effect is negative if the baseline effective capital is above its steady-state value and, therefore, the overall effect of factor substitution on per capita income, capital and consumption is ambiguous in this case. Furthermore, the steady-state labor supply is decreasing (resp., increasing) and the capital income share is increasing (resp., decreasing) in the elasticity of substitution if the baseline effective capital is below (resp., above) its steady-state value.

Klump and de La Grandville (2000) show that, in the Solow model, for two identical economies differing uniquely in the elasticity of substitution, the one with the higher elasticity of substitution will have a higher level of per capita income (and per capita

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capital) not only in the steady state but also along the transition. Thus we examine whether this last result is also true in the present model. Given the complexity of the model, an analytical treatment is probably untractable so we rely on numerical simulations. Our results show that the positive relationship between the elasticity of substitution and per capita income observed in the Solow model does not necessarily carry out to the RCK model with elastic labor supply.

Related research has been recently made. Barro and Sala-i-Martin (2004, Ch. 9) studies analytically the RCK model with endogenous labor supply, but they consider that production technology is Cobb–Douglas. Hence, they are not interested in the role of factor substitution on economic growth, which is the aim of this paper. Klump (2001) studies the effect of the elasticity of substitution on the steady-state effective capital and labor supply in a monetary growth model with elastic labor supply. However, he does not study its effect of on per capita (or aggregate) variables as income, capital and consumption. As it will be shown, this extension is not trivial. Gómez (2015) considers the one-sector endogenous growth model with physical and human capital, but labor supply is inelastic. Gómez (2017) analyzes the nexus between factor substitution and long-run growth in the Lucas (1988) endogenous growth model with human capital and endogenous labor supply. However, in this model the steady-state labor supply is independent of the elasticity of factor substitution. Furthermore, the focus in Gómez (2015, 2017) is on the effect of factor substitution on long-run growth.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the effects of the elasticity of substitution on economic performance. Section 4 concludes.

2. The neoclassical model with elastic labor supply

We consider a closed economy populated by a unit measure of identical, infinitely-lived households. The household's size N grows at the constant rate n and, for simplicity, the initial size $N(0)$ is normalized to unity. Each member of the household is endowed with one unit of time which can be allocated to work, u, or leisure, $1-u$.

2.1. Households

The representative household derives utility from (per capita) consumption C and disutility from work time u in accordance with the intertemporal utility function

$$U=\int_0^\infty\left[\ln C-\upsilon\frac{u^{1+\eta}}{1+\eta}\right]e^{-(\rho-n)t}dt,\quad \eta>0,\ \upsilon>0,\ \rho>0, \tag{1}$$

where $\rho > n$ is the rate of time preference and η is the inverse of the Frisch elasticity. The household supplies labor and rents the capital to firms producing goods. The rate of return on capital is denoted r, and the wage rate, w. The budget constraint in per capita terms is

$$\dot K = rK + wu - C - (n+\delta)K, \tag{2}$$

where K is per capita capital and δ is the depreciation rate.

2.2. Firms

Aggregate output is produced using aggregate capital NK and labor $L = uN$ by means of the CES technology

$$NY = F(NK, L) = B[\alpha(NK)^\psi + (1-\alpha)L^\psi]^{1/\psi},$$
$$B>0,\ 0<\alpha<1,\ \psi<1,$$

where Y is per capita output. Here B is the productivity parameter, α is the distribution parameter and $\sigma = 1/(1-\psi)$ is the elasticity of substitution. Let $y = Y/u$ denote the effective production; i.e., production per unit of labor, and let $k = K/u$ denote the effective capital. The production function in intensive form can be written as

$$y = f(k) = F(k,1) = B[\alpha k^\psi + (1-\alpha)]^{1/\psi}.$$

2.3. Equilibrium

A competitive equilibrium for this economy is defined as a set of market-clearing prices and quantities such that (i) the household's choice of C, u and K maximizes (1) subject to the budget constraint (2), given the initial endowment of capital, $K(0)$, and taking as given the path of factor returns, and (ii) the firms' choice of capital and labor maximizes profits.

The current-value Hamiltonian of the household's maximization problem is

$$\mathcal{H} = \ln C - \upsilon\frac{u^{1+\eta}}{1+\eta} + \lambda[rK + wu - C - (n+\delta)K],$$

where λ is the shadow value of capital. The first-order conditions are

$$\partial\mathcal{H}/\partial C = \frac{1}{C} - \lambda = 0, \tag{3}$$

$$\partial\mathcal{H}/\partial u = -\upsilon u^\eta + \lambda w = 0, \tag{4}$$

$$\dot K = rK + wu - C - (n+\delta)K, \tag{5}$$

$$\dot\lambda = (\rho-n)\lambda - \partial\mathcal{H}/\partial K = (\rho+\delta-r)\lambda, \tag{6}$$

together with the transversality condition[1]

$$\lim_{t\to\infty} e^{-(\rho-n)t}\lambda K = 0. \tag{7}$$

Profit maximization by competitive firms entails that

$$r = \frac{\partial F}{\partial(NK)}(NK, L) = f'(k) = \alpha B^\psi[f(k)/k]^{1-\psi}$$
$$= \alpha B\left[(1-\alpha)k^{-\psi} + \alpha\right]^{\frac{1-\psi}{\psi}}, \tag{8}$$

$$w = \frac{\partial F}{\partial L}(NK, L) = f(k) - kf'(k) = (1-\alpha)B^\psi f(k)^{1-\psi}$$
$$= (1-\alpha)B\left[1-\alpha+\alpha k^\psi\right]^{\frac{1-\psi}{\psi}}. \tag{9}$$

Thus, the capital income share is given by

$$\pi = \frac{kf'(k)}{f(k)}. \tag{10}$$

Using (3) and (9) to substitute for λ and w in (4) we get an expression of per capita consumption as a function of k and u:

$$C = C(k,u) = \frac{w}{\upsilon u^\eta} = \frac{f(k)-kf'(k)}{\upsilon u^\eta}. \tag{11}$$

Log-differentiating (3), using (6) and (8), we get the evolution of per capita consumption as

$$\frac{\dot C}{C} = r - \rho - \delta = f'(k) - \rho - \delta. \tag{12}$$

Substituting (8) and (9) into the budget constraint (2), we get the resources' constraint in per capita terms,

$$\dot K = uf(k) - C - (n+\delta)K = u[f(k) - (n+\delta)k] - C. \tag{13}$$

[1] These conditions are sufficient because the current-value Hamiltonian is concave in the states and the controls.