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Shape optimization of a mechanically decoupled six-axis force/torque sensor

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ABSTRACT

This paper presents the design optimization of a mechanically decoupled six-axis force/torque (F/T) sensor by minimization of cross coupling error. The new term 'principal coupling' is proposed to define the largest cross coupling error. In the first design step of the F/T sensor, the locations of twenty-four strain gages in a sensor structure are predetermined, and four structural design variables are selected to be optimized. In the second step, an optimization framework that reduces principal coupling is developed. Multiple constraints on good isotropic measurement and safety are considered and formulated using the output strain of each strain gauge circuit. The optimal design utilizes FEM software and MATLAB interactively to perform effective shape optimization. As a result of shape optimization, principal coupling of a six-axis F/T sensor was reduced from 35% to 2.5% with good isotropy. The final design of the F/T sensor was fabricated for experimental verification and there was only 0.7% difference in principal coupling and 5.2% difference in the overall strain output between the numerical and experimental results. The optimal design results in this paper are expected to provide a design guideline for multi-axis F/T sensors with significantly reduced cross coupling error, one of the biggest technical obstacles in developing F/T sensors.

1. Introduction

Six-axis F/T(force/torque) sensors are widely used in robotics applications, to measure three axial forces (*Fx*, *Fy*, *Fz*) and three axial moments (*Mx*, *My*, *Mz*) simultaneously. Because of the high price of the six-axis F/T sensor, its use has been largely confined to the robotics field, where accurate force control is required for industrial robots in particular. However, recent increasing demand for this type of sensor in automation and human-robot interaction technology in various industries and engineering researches such as biomechanics [1], sports medicine [2], humanoid robots [3,4], and medical applications [5–7] has promoted active research on the development of economical multi-axis F/T sensors.

Generally, six-axis F/T sensors can be divided into two types according to the relationship between applied force and the output signal: mechanically coupled sensors and mechanically decoupled sensors [2,8–10]. In a mechanically coupled sensor, an applied pure force component generates an output signal in more than one bridge circuit and this signal must be calibrated with a relatively complicated calibration matrix. In a mechanically decoupled sensor, on the other hand, the output signal of a bridge selectively responds to a specific force or moment component. Sensor calibration and maintenance are relatively easy compared to the case of a mechanically coupled sensor, as the output signals are physically decoupled and malfunction can be easily identified. Another benefit of a decoupled F/T sensor is that subtraction of a sensing component is possible for low cost. That is, a four or five-axis F/T sensor can be easily obtained from a six-axis F/T sensor by selectively eliminating strain gauge bridges without any structural modification. However, despite the numerous advantages of mechanically decoupled sensors, they have not been widely used in industries because of difficulty of designing them with low cross coupling and simple geometry [8].

For the development of a six-axis F/T sensor, the cross coupling is an important factor regarding sensor quality. In this paper, the cross coupling is conceptually defined as the ratio of unfavorable signal to the intended signal at a given bridge circuit according to pure force components. For example, suppose that a circuit output signal which is intended to measure force *Fx*, also response to moment *My*. In this case, cross coupling is the ratio of the output signal under the maximum *My* (unfavorable signal) to that under the maximum *Fx* (intended signal). Since a large amount of cross coupling (generally ranging from 3 to 37% [11]) adversely affects the sensor quality, many studies have been conducted on the design of a completely decoupled F/T sensor.

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Since the Maltese cross elastic element was first introduced [12], most six-axis force torque sensors have been developed based on this element with four elastic beams crosswise [8]. The conventional Maltese cross element having rigid inner and outer flanges is effective for measuring some force components but has low sensitivity to horizontal forces (*Fx* and *Fy*) and large cross coupling [9]. Various modifications have thus been carried out to increase the sensitivity and reduce cross coupling, such as by addition of a ball bearing at the end of an elastic beam [8], installation of elastic sliding spherical joints [13], and application of a parallel plate structure (PPS) [14,15] or plate spring at the end of elastic beams [16–18] for greater deformation and higher sensitivity. However, the ad hoc selection and modification of design variables in previous studies does not guarantee a sensor design with global minimal error and is not time efficient.

Some researchers have conducted structural optimization with design variables and constraints to overcome these limitations. Quinn and Mote [19] introduced a mathematical modeling and design optimization of an uncoupled six-degree-of-freedom dynamometer and Ma et al. [20] suggested analytical solutions for fast estimation of strain distribution of cross-beam six-axis F/T sensor. Their methods provide insight into sensor analysis but require different sensor structures, which are difficult to mathematically analyze. Bicchi [21] proposed a mathematical objective function to optimize sensor accuracy but this method is related to the number and the position of the basic transducers, not to the geometric dimensions of sensor structure. A convenient numerical shape optimal design technique was suggested by Chao [8] and Liu [9]. However, they did not consider coupling error in the design procedure and their relatively simple objective functions (minimizing weight and volume of the sensor) make it hard to control precise design requirements such as strain values on designated points. Hayashi improved the optimization algorithm including singular values of the compliance matrix in the objective function by using MATLAB and FEM together [22], and some other groups further suggested algorithms with different design of a multi-axis F/T sensor [23–25], but none considered coupling error. Therefore, although previous works yielded improved multi-axis F/T sensor designs, the outcomes possess high degrees of cross coupling errors [9,14,22].

Some studies on multi-axis F/T sensors have reported low coupling terms, but most considered too small moment to force specification: 20 Nm/392 N = 0.051 Nm/N [8,9], 10.5 Nm/350 N = 0.03 Nm/N [11], 2.5 Nm/200 N = 0.0125 Nm/N [14], and 0.09 Nm/20 N = 0.0045 Nm/N [26]. However, our observation shows coupling error becomes larger as moment to axial force specification increases.

For example, suppose that there is a sensor whose moment to force specification is 20 Nm/400 N = 0.05 Nm/N and certain bridge circuit of the sensor is required to generate the strain 1000 μm/m under $F_x^{\max} = 400\,\mathrm{N}$ (intended signal). But the bridge circuit also produces 10 μm/m under maximum moment $M_y^{\max} = 20\,\mathrm{Nm}$ (unfavorable signal). In this case, coupling error becomes 10/1000 × 100 = 1%. Let's assume that this sensor can be used in the different force and moment range ($F_x^{\max} = 400N$, $M_y^{\max} = 40Nm$). Then the moment to force specification is doubled as 40 Nm/400 N = 0.1 Nm/N as well as the unfavorable signal (20 μm/m), ending up with twice larger coupling error (20/1000 × 100 = 2%). So, by considering smaller moment to force specification, some of previous works possibly ignored significant coupling. To summarize, the moment to force specification should be included for the fair comparison of cross coupling. This paper considers a relative high moment to force specification for practical use of sensor (e.g., 0.1 Nm/N).

This paper proposes a numerical shape optimization design procedure with effective representation and minimization of the cross coupling term, which is a practically important for the performance of mechanically decoupled six-axis F/T sensors. The target load specifications are *Fx* = *Fy* = 400 N, *Fz* = 800 N, and *Mx* = *My* = *Mz* = 40 Nm (moment to force specification = 0.1), considering practical applications to full size humanoid robots [3] and in human-robot interaction [5,27] such as haptic devices used for human force augmentation. The organization of this paper is as follows. In Section 2, cross coupling is explained from the relationship between applied load and output strain, and a new definition of principal coupling is presented. Section 3 explains the sensor structure and suitable positions of strain gauges. In Section 4, design parameters are defined, and an optimization problem is formulated to minimize principal coupling while securing safety and good sensitivity. Interactive utilization of the FEM software MATLAB is employed to conduct optimization and the final design result is manufactured for verification. The validity of the optimized design is demonstrated experimentally in Section 5.

2. Performance criteria of six-axis F/T sensors

2.1. Relationship between applied load and output strain

Consider the six-axis F/T sensor under a certain load $\vec{F} = \left[F_x, F_y, F_z, M_x, M_y, M_z\right]^T$ applied on the center within its linear elastic range. The resultant strain output generated in n Wheatstone bridges can be written as a $n \times 1$ strain output vector $\vec{S}$. When the sensor behaves under the elastic range, the relationship between the load vector and the strain output vector can be written as:

$$\vec{S} = [C]\vec{F}, \tag{1}$$

where $[C]$ is an $n \times 6$ strain compliance matrix whose element C_{ij} represents the strain contribution at bridge circuit i due to a unit pure load j (this paper considers $n = 6$). When the maximum pure load for x direction is applied ($\vec{F}_1 = [F_x^{max} 0\,0\,0\,0\,0]^T$), the corresponding strain output vector is: $\vec{S}_1 \equiv [S_{11}S_{21}\ldots S_{61}]^T = [C]\vec{F}_1 = F_x^{max} \times [C_{11}C_{21}\ldots C_{61}]^T$ and only C_{ij} for $j = 1$ becomes active. This relationship for the other maximum pure load components reads as follows:

$$\begin{aligned}
\vec{S}_2 &= F_y^{\max} \times [C_{12}C_{22}\ldots C_{62}]^T \\
\vec{S}_3 &= F_z^{\max} \times [C_{13}C_{23}\ldots C_{63}]^T \\
\vec{S}_4 &= M_x^{\max} \times [C_{14}C_{24}\ldots C_{64}]^T \\
\vec{S}_5 &= M_y^{\max} \times [C_{15}C_{25}\ldots C_{65}]^T \\
\vec{S}_6 &= M_z^{\max} \times [C_{16}C_{26}\ldots C_{66}]^T
\end{aligned} \tag{2}$$

The strain matrix, ***S***, can be defined by combining six $\vec{S}_j$ column vectors as follows:

$$S = \left[S_{ij}\right] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{bmatrix} \tag{3}$$

The matrix in Eq. (3) is used for the definitions of cross coupling and principal coupling in the next section.

2.2. Cross coupling and principal coupling

If the strain output matrix in Eq. (3) is a diagonal matrix with all zero off-diagonal values, then the sensor is considered completely decoupled. However, when one bridge circuit responds to more