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Multi-modal vibration energy harvesting utilizing spiral cantilever with magnetic coupling

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ABSTRACT

Most vibration energy harvesters use conventional cantilever configuration and typically perform well for a specific frequency at the first resonant mode, this makes the devices less effective in ambient vibrations with varying frequencies. This paper presents a multi-modal vibration energy harvester achieving multiple peaks in the frequency response and causing the possibility of widening the operation frequency range. A spiral-shaped cantilever with tip mass in the form of magnets coupling with a magnetoelectric (ME) transducer is adopted in this harvester. The spiral-shaped beam is shown to be conductive to presenting multi-modal responses and lowering the natural frequencies of the harvester. Furthermore, due to the magnetic coupling between the magnets and the transducer, the peak frequencies are tunable and the frequency spacing between the adjacent modes can be obviously reduced. The operating principle of energy conversion is based on the relative movement of the magnets and the transducer, and the effects of magnetic coupling working on the peak frequencies are experimentally determined. The experimental results indicate that the proposed harvester can obtain five obvious peak values in the range of 15–70 Hz, which are concentrated around 20.7, 26.1, 32.3, 42.2 and 63.7 Hz, respectively.

1. Introduction

Harvesting energy from ambient energy sources offers an effective method to power the small-scale systems, and environmental vibration energy is a very promising alternative energy source for its ubiquitous existence. Many efforts have recently been made to research and develop the vibration energy harvesters based on various energy conversion mechanisms [1–4], which achieve the optimum energy generation when operating around or at their first mode resonant frequencies. If the ambient vibration excitation deviates from the resonant condition, the electric output will reduce significantly and is too low to be utilized. To address this problem, several solutions have been developed in the literature such as employing active or passive resonant frequency tuning techniques [5–11], or increasing the operating bandwidth of the harvester. Some of these frequency tuning methods are implemented manually, or require extra structures or energy. Widening the working frequency region solution has also been proposed by many researchers [12–14], although the bandwidth of the harvester is increased through using series of beams, only one single beam at a time contributes to the power generation for a given frequency, which actually reduces the output power per unit volume.

Recently, researchers have proposed a new promising approach to improve the operating condition of the harvesters by harvesting energy from multiple vibration modes of the structure. Yang et al. [15] investigated an electromagnetic harvester based on a suspended beam with three permanent magnets, and the proposed design can harvest energy under three environment vibration frequencies of 369, 938 and 1184 Hz, respectively. Suzuki et al. [16] exploited an asymmetric gammadion spring as the vibration resonator to achieve multiple frequency energy harvesting corresponding to the resonant frequencies of 110, 165 and 243 Hz, respectively. Since the frequencies of typical ambient vibrations are lower than 100 Hz, it is more meaningful to achieve multiple modes in this low frequency region. A V-shaped energy harvester with multiple magnets was presented in [17], the geometries of the V-shaped plate and the locations of the magnets were analyzed to control the resonant frequencies and their spacing. This harvester can obtain three natural frequencies in the range from 8 to 19 Hz, but needs multiple energy conversion structures (multiple sets of magnets and coils). The device proposed in 2012 [18], which consisted of a conventional cantilever positioned on a flexible body beam, was shown to be able to harvest energy from the first three vibration modes of the device with a large frequency spacing between the first mode (at 8.7 Hz) and the second mode (at 55.8 Hz). The L-shaped beam-mass structure investigated in [19] could achieve harvesting energy from the first two vibration modes with $\omega_2 \approx 2\omega_1$, while the third and the higher modes are far removed from the first two modes.

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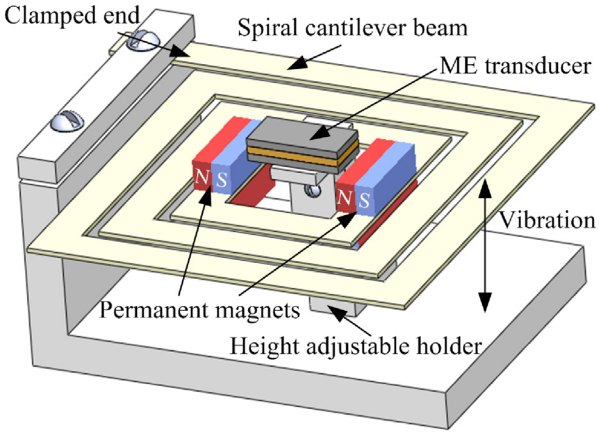

Fig. 1. Schematic diagram of the vibration energy harvester.

Aiming to achieve more vibration modes in the low frequency range, we utilize a single magnetic spiral-shaped cantilever to harvest vibration energy from the first five modes of the structure. The spiral shape configuration contributes to exhibiting multi-modal responses for the bending and torsion vibration. Moreover, the magnets acting as the proof mass of the spiral beam interacts with a ME transducer, and the magnetic force between them affects the vibration of beam and alters the modal frequencies with a smaller frequency spacing. The prototype harvester is fabricated and experimentally tested, and the vibration (the bending and the torsion vibration) both without (moving away the transducer) and with the magnetic coupling is tested to investigate the effect of the magnetic coupling.

2. Description of the harvester

A schematic diagram of the proposed multi-modal vibration energy harvester is shown in Fig. 1, which consists of a spiral-shaped cantilever beam, four permanent magnets, a ME transducer and a height adjustable holder. The spiral cantilever with the same width and thickness acts as the resonator of the harvester, and four permanent magnets are arranged and positioned at the innermost layer of the spiral beam acting as the proof mass. The ME transducer fixed on the holder is fabricated by one piezoelectric layer bonded between two magnetostrictive layers. Both the spiral beam and the holder are fixed on the housing of the harvester, and the energy conversion is achieved based on the relative movement of the magnets and the ME transducer.

3. Working principle

3.1. Spiral cantilever design

The resonator of this harvester is designed as a spiral-shaped beam, which is fixed at one end and acts as a cantilever structure with tip mass in the form of permanent magnets. Here, the spiral cantilever is modeled as a series of beam segments with the same width and thickness, and the analytical model is shown in Fig. 2. The structure parameters w, s and t shown in Fig. 2, respectively denotes the width of spiral segments, spacing between adjacent spiral layers and the thickness of the beam.

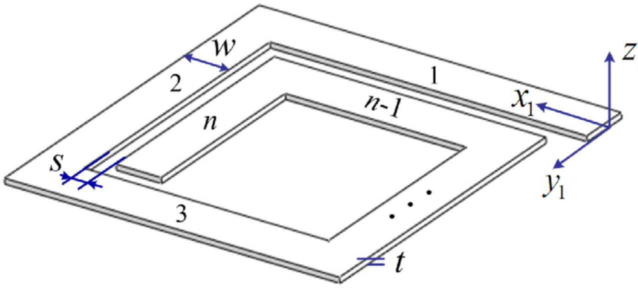

Fig. 2. Analytical model of the spiral cantilever.

As shown in Fig. 2, each beam segment is connected to the adjacent beams at its two ends, both the bending motion and torsion of each beam segment will result in the next beam move out of the main plane. The governing equations of bending and torsion for the free vibration of each beam segment (with damping neglected) can be written as

$$EI\frac{\partial^4 z_i(x_i,t)}{\partial {x_i}^4} + \rho A\frac{\partial^2 z_i(x_i,t)}{\partial t^2} = 0 \tag{1}$$

$$GJ\frac{\partial^2 \gamma_i(x_i,t)}{\partial {x_i}^2} - I_M\frac{\partial^2 \gamma_i(x_i,t)}{\partial t^2} = 0 \tag{2}$$

where $z_i(x_i,t)$ is the deflection of the beam segment i $(i=1\sim n)$, E is the Young's modulus, $I = wt^3/12$ is the area moment of inertia, ρ and $A = wt$, respectively denotes the density and the cross section area of the beam segment, γ_i is the twist angle, $GJ = Gw^3t^3/3(w^2+t^2)$ is the torsional rigidity, $I_M = \rho A(w^2+t^2)/12$ is the mass moment of inertia. Eqs. (1) and (2) can be solved by using the separation of variables method with general solution respectively expressed as $z_i(x_i,t)=\phi_i(x_i)e^{i\omega_n t}$ and $\gamma_i(x_i,t)=\beta_i(x_i)e^{i\omega_n t}$, ω_n is the natural frequency. Here, the analysis procedure is performed to obtain the natural frequencies of the spiral beam without the magnetic coupling. Therefore, the transducer is moved away, and the magnets are simply considered and modeled as a tip mass at the free end of the beam. Then the boundary conditions at the clamped end ($x_1=0$) and the free end ($x_n=l_n$) can be written as

$$\phi_1(0)=0,\quad \phi_1'\big|_{x_1=0}=0,\quad \beta_1(0)=0,$$
$$EI\phi_n''\big|_{x_n=l_n}=0,\quad \beta_n'\big|_{x_n=l_n}=0,\quad EI\phi_n''\big|_{x_n=l_n}=-m\omega_n^2 \tag{3}$$

where m is the equivalent mass of the magnets, and l_n is the length of the last beam member and considered to leave appropriate room for the magnets. Then, the length of the beam segment i (l_i) can be given by

$$\begin{cases} l_i = l_n + (n-i)(w+s)/2 \quad (i = n-2k,\quad k=0,1,2,\ldots) \\ l_i = l_n + (n-i-1)(w+s)/2 \quad (i = n-(2k+1),\quad k=0,1,2,\ldots) \end{cases} \tag{4}$$

As each of the beam elements are connected to the adjacent beam member either at their beginning ($x_i=0$) or at their end ($x_i=l_i$). Therefore, the continuity condition between the neighboring beam segments can be given by

$$\phi_i(0)=\phi_{i-1}(l_{i-1}),\quad \phi_i'\big|_{x_i=0}=\phi_{i-1}'\big|_{x_{i-1}=l_{i-1}}\quad EI\phi_i''\big|_{x_i=0}$$
$$=EI\phi_{i-1}''\big|_{x_{i-1}=l_{i-1}}\quad \beta_i(0)=\beta_{i-1}(l_{i-1}),\quad GJ\beta_i'\big|_{x_i=0}=GJ\beta_i'\big|_{x_{i-1}=l_{i-1}} \tag{5}$$

According to the boundary and the continuity conditions given in Eqs. (3) and (5), the mathematical relation of the last beam segment to the first one can be obtained, which is a function of the structure parameters and the natural frequency. As the geometry parameters are decided, the natural frequencies of the spiral