Mathematical Social Sciences 93 (2018) 153–158

Contents lists available at ScienceDirect

Mathematical Social Sciences

journal homepage: www.elsevier.com/locate/mss

Gaming the deferred acceptance when message spaces are restricted[☆]

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HIGHLIGHTS

- We analyze school choice problems when students' preference lists are restricted.
- Schools' priorities are substitutable, a weaker requirement than responsiveness.
- The DA mechanism is not strategy-proof, inducing unstable assignments.
- Substitutability is more plausible than responsiveness but too weak in this problem.
- We identify the set of priorities where desirable assignments are implemented in NE.

ARTICLE INFO

Article history:
Received 28 October 2015
Received in revised form 3 October 2017
Accepted 20 March 2018
Available online 28 March 2018

ABSTRACT

We consider school choice problems where students may submit only restricted length of preference lists. We propose an acyclicity condition for the priority structure of schools. When schools' priorities are substitutable, we show that a Pareto efficient and stable assignment rule is Nash implementable by the deferred acceptance mechanism if and only if schools' priority structures are acyclic in our sense.

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1. Introduction

A school choice problem consists of five components: a set of students and a set of schools in a specified area, a profile of student preferences over schools, a profile of schools' priority rankings over sets of students, and a fixed number of school quotas. Students submit their preference lists to a central clearinghouse, and the clearinghouse uses some rule for assigning students to schools given priority rankings and the number of quotas of schools.

Many school districts use the deferred acceptance mechanism, which was introduced by Gale and Shapley (1962). The deferred acceptance mechanism performs well for school choices, to find the student optimal stable assignment (it is an assignment that no one has justified envy for the others' placement schools. Furthermore, such an assignment is not Pareto dominated among those assignments). In addition, Dubins and Freedman (1981) showed its strategy-proofness, that is, it is a dominant strategy for each student to submit her true preferences.

These powerful features are, however, crucially dependent on the assumption that each student can submit a list of preferences of any length. It is doubtful whether this assumption is appropriate for the school choice problems in reality. It is not practical for the authority to process a very long list of preferences to begin with. For instance, more than 400 New York City schools allow students to list at most 12 programs, from about 700 programs, on an application form. Moreover, it is very costly for a student to state preferences over an extremely large number of schools, and so such a restriction on the length is practical for students as well.

On the other hand, a restriction on the length affects the strategic property of the deferred acceptance mechanism. As Haeringer and Klijn (2009) first pointed out, the truth-telling strategy is no longer a dominant strategy for each student, and in fact there is no dominant strategy, under such a restriction. Thus students have incentives to behave strategically in this environment. It is then natural to ask whether the student optimal stable assignment is still obtained as a Nash equilibrium outcome or not. This is the main issue this paper tries to address.

As mentioned above, Haeringer and Klijn (2009) first studied a model with these restrictions, and they showed that the deferred acceptance mechanism yields a stable and Pareto efficient assignment in a Nash equilibrium when schools' priority rankings are strongly X-acyclic. However, their analysis assumes that a school's priorities over students must be responsive: if student i is ranked higher than another student i', then for any set N of students not containing i and i', $N \cup \{i\}$ must be ranked higher than $N \cup \{i'\}$. In words, rankings are independent of students' common colleagues' characters. We believe that this assumption

☆ This paper is based on my bachelor's thesis at Yokohama National University. I am especially grateful to Atsushi Kajii, Taro Kumano, and two anonymous referees for very helpful discussions and comments. I am also grateful to Taro Akiyama, Masahito Kobayashi, Morimitsu Kurino, Nozomu Muto, Katsumasa Nishide, Norio Takeoka, Ryuji Sano, and Hiroki Sasaki for their encouragement. All remaining errors are mine.

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https://doi.org/10.1016/j.mathsocsci.2018.03.007
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is too restrictive. Abdulkadiroğlu and Sönmez (2003) pointed out that priorities we observe in practice are often not responsive. For instance, as Kumano (2009)'s example, imagine that a school evaluates a group of students not only by their test scores but also its race composition for the sake of fair admission. Then resulting priority ranking over sets of students will not be responsive, since the race composition of classmates matters. Thus it is important to consider schools' priority rankings accommodating the example.

In this paper, we study a broader class of priority rankings which might not be responsive. We will still require that a school's priority ranking induces a choice rule which exhibits substitutability, initially introduced by Kelso and Crawford (1982) and Hatfield and Milgrom (2005).[1] Roughly speaking, it means that a student chosen by a school from a pool of applicants will also be chosen by the school from any subset of the pool of applicants including the student. It can be shown that a responsive priority ranking of a school induces a choice rule with substitutability, but not vice versa. Although the priority rankings in Kumano (2009)'s example are not responsive they can still induce a choice rule with substitutability, thus they are accommodated in our setup. So we believe that this is a setup which is broad enough to address various concerns in school choice problems in reality.

Our results concern the properties of Nash equilibrium outcomes, focusing on stability and Pareto efficiency, when students are allowed to submit a list of preferences with some restricted length in the environment described above. For stability, as a preliminary result, we first show that the assignment generated by the so-called Boston mechanism is implementable in Nash equilibrium (Proposition 1), reinforcing Kojima (2008). On the other hand, the assignment generated by the deferred acceptance mechanism is not in general. But if priority rankings are further restricted to those satisfying acyclicity of Kumano (2009), it is implementable in Nash equilibrium (Proposition 3).

As is known, the stable assignment generated by the Boston mechanism or the deferred acceptance mechanism in Nash equilibrium is not necessarily Pareto efficient in general, so we ask when a stable and Pareto efficient assignment is Nash implementable by such the mechanisms. Our main result (Theorem 1) shows that this is the case if and only if the priority rankings exhibit a condition stronger than acyclicity, which we call *strict acyclicity*.

Related literature

School districts commonly use the deferred acceptance mechanism, the top trading cycles mechanism, or the Boston mechanism. As noted, the deferred acceptance mechanism is stable and strategy-proof but not Pareto efficient. The top trading cycles mechanism was introduced by Abdulkadiroğlu and Sönmez (2003) as another way for assigning students to public schools in areas outside their resident districts. Though it is Pareto efficient and strategy-proof, the resulting assignment is not generally stable. Also the Boston mechanism, which is the assignment rule used in Boston until 2006, is Pareto efficient but neither stable nor strategy-proof. Each mechanism has a common shortcoming, that is, none is simultaneously stable, Pareto efficient, and strategy-proof.

When each school's priority ranking is responsive, Ergin (2002) characterized Pareto efficiency of the deferred acceptance mechanism by a condition on priorities called Ergin-acyclicity. Kesten (2006) and Kumano (2013) also found conditions where the top trading cycles mechanism and the Boston mechanism satisfy the three desirable properties. Those are called Kesten-acyclicity and strong acyclicity, respectively.

Haeringer and Klijn (2009) investigated the school choice problem where students are restricted in the length of preference lists. They showed that a stable and Pareto efficient assignment is obtained as a Nash equilibrium of the deferred acceptance mechanism and the Boston mechanism if and only if schools' priority rankings are strongly X-acyclic. The strong X-acyclicity implies Ergin-acyclicity.

When each school's priority ranking induces a choice function with substitutability, Kumano (2009) extended the results of Ergin (2002), and showed that the deferred acceptance mechanism is Pareto efficient if and only if priority rankings are acyclic. Our strict acyclicity condition in Theorem 1 implies Kumano (2009)'s acyclicity. The relation is analogous to the strong X-acyclicity implying Ergin-acyclicity.

2. Model

2.1. School choice problems

The sets of students and schools are denoted by N and C, respectively, which are assumed to be finite. Each student $i \in N$ has a strict preference relation P_i over $C \cup \{i\}$. Let R_i be the weak order corresponding to P_i. Denote by $\mathcal{P}_i$ the set of all such preferences for student i, and set $\mathcal{P} = \times_{i\in N}\mathcal{P}_i$. We write $P = (P_i)_{i\in N} \in \mathcal{P}$ for a profile of students' preferences.

If $cP_i i$, we say that school c is **acceptable** for student i. Denote by $|P_i|$ the number of acceptable schools, and denote by $\text{top}(P_i)$ the highest ranked school under P_i. A student will never be assigned to a non-acceptable school, so to describe a preference relation, we will just write the ordering over acceptable schools for a student. For instance, we write "$P_i : c, c'$ " to mean that cP_ic' and $c'P_ii$, and iP_ic'' for $c'' \neq c, c'$, and in this case $|P_i| = 2$ and $\text{top}(P_i) = c$. We write "$P_i : \emptyset$" to mean that there is no acceptable school under P_i.

Each school $c \in C$ has a capacity of seats, q_c, and a strict linear order $\succ_c$, which denotes the priority of c over 2^N. Let $\succsim_c$ be the weak order corresponding to $\succ_c$. A tuple of linear order and capacity is called a **priority structure** for school c, which is denoted by $(\succ_c, q_c)$. We write $(\succ, q) = (\succ_c, q_c)_{c\in C}$ for a profile of priority structures. Given a priority structure $(\succ_c, q_c)$ and the set of students $N' \subseteq N$, the induced choice function, $Ch_c : 2^N \to 2^N$, chooses the highest ranked group of students from N' according to its priority structure; that is, $Ch_c(N') \subseteq N'$, $\left|Ch_c(N')\right| \leq q_c$, and $Ch_c(N') \succsim_c N''$ for any $N'' \subseteq N'$ with $\left|N''\right| \leq q_c$.

The induced choice function Ch_c exhibits **substitutability** if for any $N'' \subseteq N' \subseteq N$, $Ch_c(N') \cap N'' \subseteq Ch_c(N'')$ holds. Note that if Ch_c exhibits substitutability, it is **path independent**, *i.e.* , $Ch_c(Ch_s(N') \cup N'') = Ch_c(N' \cup N'')$ for any $N', N'' \subseteq N$. Ch_c exhibits **acceptance** if $\left|Ch_c(N')\right| = \min\left\{q_c, \left|N'\right|\right\}$ holds for any $N' \subseteq N$. A priority structure is said to be substitutable and acceptant if it induces a choice function which exhibits the two properties. Throughout this paper, we assume that a priority structure is substitutable and acceptant for all schools. We note that in the school choice context, a stronger condition is often assumed: a priority structure is said to be **responsive** if for any $i, i' \in N$ and for any $N' \subseteq N \setminus \left\{i, i'\right\}$, $N' \cup \{i\} \succ_c N' \cup \left\{i'\right\}$ holds if and only if $\{i\} \succ_c \left\{i'\right\}$.

For two sets of students N' and N'' with $N'' \subseteq N' \subseteq N$ and $\left|N''\right| \leq q_c$, let $Ch_c(N'|N'')$ be the set of students who are chosen by school c from N' when the students in N'' must be included, *i.e.* , $N'' \subseteq Ch_c(N'|N'') \subseteq N'$, $\left|Ch_c(N'|N'')\right| \leq q_c$ and $Ch_c(N'|N'') \succsim_c M$ for any M with $N'' \subseteq M \subseteq N'$ and $|M| \leq q_c$.

An **assignment** μ is a mapping from N to $C \cup N$ such that for every $i \in N$, $\mu(i) \in C \cup \{i\}$ and $|\mu(i)| = 1$, and for every $c \in C$, $\left|\mu^{-1}(c)\right| \leq q_c$. An assignment μ is said to be **individually rational** if for every $i \in N$, $\mu(i)R_ii$ and $\mu^{-1}(c) = Ch_c(\mu^{-1}(c))$ hold. A

[1] The reason we consider substitutability priority is that it is more general than widely used domain of responsiveness, and the existence of stable assignments is guaranteed (cf. Roth and Sotomayor, 1990).

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