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Collective identity functions with status quo[☆]

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HIGHLIGHTS

- We study self-determination to a group, characterized by one attribute.
- We admit the possibility that opinions are incomplete.
- A status quo option can intervene in the process.
- We define ternary collective identity functions with *status quo*.
- We give some axiomatic characterizations of ternary collective identity functions.

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ABSTRACT

In this paper, we define the problem of group identification in an extended environment. We concentrate on the problem where the society is required to self-determine the belongingness of each member to a specific group, characterized by a single attribute. In general terms, this case consists of a collective identity issue that can be regarded as an aggregation problem of individual assessments within a group. Here we introduce the possibility that opinions are incomplete and that a *status quo* option intervenes in the process.

1. Introduction

We study aggregation rules in which each individual in a society proposes a subgroup of members and a subgroup of non-members of the society and, given the profile of proposed subsets and a *status quo* subgroup of members, the rule produces the list of qualified individuals.

Our model is a modification of the model that Kasher (1993) first posed from a philosophical perspective, namely, the collective identity question. From a formal point of view, Kasher and Rubinstein (1997) draw a bridge between the idea of collective identity and the algebraic theory of aggregators.

In their first model each individual in a society proposes a subgroup of members of the society and the rule produces the subgroup of qualified individuals. They provide axiomatic characterizations for three extreme aggregation rules: the strong liberal, the dictatorships (under some technical assumptions, see also Saporiti, 2012), and the oligarchical rules. In their study of consent rules, a class of voting rules with aspects of majoritarianism and liberalism, Samet and Schmeidler (2003) follow Kasher and Rubinstein to study the relation between the liberal and the majoritarian rules. Sziklai (2015) proposes and axiomatizes an algorithm based on the so-called top candidate relation. He also demonstrates its effectiveness with a case study that uses citation data.

We can cite other approaches to the endogenous classification problem. Besides the aforementioned description, Kasher and Rubinstein also considered a second model where every person expresses her opinion about how the society should be partitioned into non-ranked classes. A decomposition of the group into classes from such profiles should be derived. Houy (2007) and Miller (2008) among others have been concerned with the case in which the number of classes is fixed (in the collective identity problem this number is two). Other authors have studied the case in which the individuals only express their wishes about who should be put together in the same classes, e.g., Kasher and Rubinstein (1997), Houy (2007) and Dimitrov and Puppe (2011). Dimitrov (2011) is a nice survey of articles about the group identification problem, which includes further references like Ballester and García-Lapresta (2008), Billot (2003), Çengelci and Sanver

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(2010), or Dimitrov et al. (2007). More recently, Cho and Ju (2017) provide a theoretical foundation for the liberal rule, i.e., the self-identification method commonly used for racial or ethnic classifications. Cho and Ju (2015) unify the models by Miller (2008) and Cho and Ju (2017) in an extended framework that uncovers implicit constraints in them. Furthermore, Alcantud and de Andrés (2017) provide a richer environment that captures the subjectivity associated with general and vague attributes in the form of partial memberships. It builds on the fact that the modeling of how individuals in a society are collectively viewed as belonging to a group is often associated with vague attributes like 'belonging to a newly formed nationality' (Dimitrov et al., 2007), 'being Jew' (Kasher, 1993), 'being African-American' or belonging to any racial group (Miller, 2008), or 'living in a rich neighborhood' (Dimitrov, 2011). Erdélyi et al. (2017) analyze the complexity of bribery and destructive control in group identification, with emphasis on the cases of consent rules, the consensus-start-respecting rule, and the liberal-start-respecting rule. Finally, strategic considerations are the subject of recent interesting analyses like Cho and Saporiti (2017) or Yang and Dimitrov (2016).

The present paper extends the standard collective identity problem in two directions: firstly, voters are allowed to abstain on the others' identities, and secondly, there exists an initial group of *status quo* members. Because people can abstain, each person is asked to propose a subgroup of members and a subgroup of non-members of the society. Relatedly, Alcantud and Laruelle (2016) give an extensive analysis of ternary trichotomous voting rules. This feature has been implemented in this literature by Ju (2010), who characterizes various families of rules represented by systems of powers. The exogenously given subgroup of *status quo* members permits several interpretations. Sziklai (2015) studies a model where some individuals are allowed to form an opinion but they are not elective. As an example, the award of grants among the members of a society can rule out recipients of grants in the past two years although everyone can voice their opinion. This possibility can easily be subsumed in our model, which might also capture the idea that we are exploring a situation where a group revises its structure. The *status quo* group can also characterize some members of the society through a different (either positive or negative) aspect like seniority, mainstreamness or professionalism. The idea is that experts should identify each other better, whereas dilettantes or laypersons can be led to recommend popular dabblers and discard the real innovators.[1] Therefore a distinctive feature can help to fine-tune the final selection. Remarkably, the *status quo* group could also define a default selection like the threat or disagreement points in the cooperative bargaining problem. For example, in case of lack of agreement on a renewed board of advisors, the two older members are chosen. The latter possibility suggests the term *status quo* to refer to that subgroup.[2]

So the question is who should be a member of the collective identity given that a subgroup of the population is exogenously given (under one of the many possible interpretations that we have hinted) and that the whole population express their opinion about the issue with the possibility of abstention. To describe this situation we put forward a model of ternary collective identity functions that includes the features we have reported. We outline a number of examples with different characteristics. Then we explore some axioms adapted to our setting, and finally we produce some characterizations of ternary collective identity functions.

Tables 1 and 2 summarize the achievements of earlier axiomatic approaches to the group identification problem. Following Kasher and Rubinstein (1997), we distinguish two types of analyses. In Table 1, the question is who belongs to the group? Therefore one single attribute determines belongingness, be it defined by a race, a nationality, or any other characteristic. In Table 2, the question is how do we split the society into subgroups? In each instance, we list the criteria that each reference characterizes. In cases $*$ further technical restrictions are imposed.

In only one case there is a selected subgroup that might affect the final decision. This is Sziklai (2015), who proposes a model with a subgroup of non-elective members. His interpretation is one of the possible uses of our *status quo* group.

The organization of the paper is as follows. Section 2 introduces the relevant terminology and definitions. In particular, we define the notion of ternary collective identity functions with status quo. We also propose a list of noteworthy examples of rules, define properties that the rules might satisfy, and check if our examples satisfy them. Section 3 contains some axiomatic characterizations of ternary collective identity functions. We end up with some concluding remarks.

2. Ternary collective identity functions with status quo

Let $N = \{1, \ldots, n\}$ be the set of agents who face the problem of collectively choosing a subset of N. Our model has two inputs.

The first input is the initial subset of members: $M \subseteq N$. The meaning of this subset is subject to interpretations, some of which we discussed in Section 1. For convenience, we refer to M as the status quo (SQ) in the input, and members of M are named SQ-members. We denote by $\mathcal{P}(N)$ the set of subsets of the society N.

The second input is a matrix of ternary opinions, that we denote A. This is a square matrix $A = (a_{ij})_{i,j \in N}$, such that $a_{ij} \in \{-1, 0, 1\}$. The element a_{ij} represents i's opinion on j's membership in the group identity: $a_{ij} = 1$ means that i considers that j should be a member, $a_{ij} = -1$ means that i considers that j should not be a member, and when $a_{ij} = 0$ we interpret that i abstains on j's membership. The ith row represents agent i's opinion on the n members of the society, while the jth column represents the n agents' opinions on person j. We denote by $\mathcal{T}$ the set of matrices with the above described format. We also say that $A \in \mathcal{T}$ is a profile of (ternary) opinions.

A subset of N must be chosen. Such choice depends on both the opinion of the whole population about who should belong to it and the initial subset.

[1] A recurring example of such behavior is the choice of FIFA Ballon d'Or or Best FIFA Men's Player. In 2012, the winning combination of FIFA Ballon d'Or was Lionel Messi (1st), Cristiano Ronaldo (2nd), and Andrés Iniesta (3rd). Out of 162 national team captains that voted, only 15 chose this combination, namely, the captains of Antigua and Barbuda, Azerbaijan, Bhutan, Bulgaria, Cayman Islands, Chad, Faroe Islands, Moldova, Mongolia, Northern Ireland, Poland, Republic of Ireland, Sao Tome e Principe, Trinidad and Tobago, and Vietnam. Among the 162 coaches that voted, only 19 voted for the winning combination, which were the coaches of Bangladesh, Barbados, Burkina Faso, Burundi, Cameroon, Chad, Czech Republic, Eritrea, Guinea, Jordan, Korea Republic, Moldova, Myanmar, Poland, Slovakia, Swaziland, Trinidad and Tobago, and Uruguay. Source: http://bleacherreport.com/articles/1475528-fifa-ballon-dor-voting-analysis-one-for-the-stat-geeks. Accessed on April 3rd, 2017. In 2016, the voting for the Best FIFA Men's Player marks a change to the original format. It encompasses four separate groups, the general public now being allowed to cast their vote (alongside more professional experts: media representatives, national team coaches, and national team captains). Each group has 25 per cent of the overall vote. Source: http://www.dailymail.co.uk/sport/football/article-4025566/Cristiano-Ronaldo-goes-against-Lionel-Messi-Ballon-d-Best-FIFA-Men-s-Player-Award-come-s-difference.html Accessed on April 3rd, 2017.

[2] Cho and Ju (2015) use the same term in a very particular framework. Their setup is a multinary model. In passing they explain that none of the consent rules except for the liberal rule is well-defined in their model. The reason is that with insufficient consent, an agent fails to self-determine her membership and the result is indeterminate because there are multiple remaining groups. To avoid this drawback one can introduce a status quo group in such way that each person either belongs to the group of her choice or the status quo group (cf., Cho and Ju, 2015 Section 5.2.2).