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Short communication

A Simple optimality-based no-bubble theorem for deterministic sequential economies with strictly monotone preferences☆

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HIGHLIGHTS

- A simple no-bubble theorem that applies to a wide range of deterministic sequential economies with infinitely lived agents is established.
- The theorem shows that asset bubbles never arise if at least one agent can reduce his asset holdings permanently from some period onward.
- The theorem is based on the optimal behavior of a single agent, requiring virtually no assumption beyond the strict monotonicity of preferences.

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ABSTRACT

We establish a simple no-bubble theorem that applies to a wide range of deterministic sequential economies with infinitely lived agents. In particular, we show that asset bubbles never arise if at least one agent can reduce his asset holdings permanently from some period onward. Our no-bubble theorem is based on the optimal behavior of a single agent, requiring virtually no assumption beyond the strict monotonicity of preferences. The theorem is a substantial generalization of Kocherlakota's (1992) result on asset bubbles and short sales constraints.

1. Introduction

Since the global financial crisis of 2007–2008, there has been a surge of interest in rational asset pricing bubbles, or simply "asset bubbles". Numerous economic mechanisms that give rise to asset bubbles are still being proposed. In constructing models of asset bubbles, it is important to understand the conditions under which asset bubbles do or do not exist. While the conditions for the existence of bubbles are mostly restricted to specific models, some general conditions for nonexistence are known.

Most of the results on the nonexistence of bubbles, or no-bubble theorems, for general equilibrium models can be grouped into two categories. A no-bubble theorem of the first category typically states that asset bubbles never arise if the present value of the aggregate endowment process is finite. Wilson's (1981, Theorem 2) result on the existence of a competitive equilibrium can be viewed as an earlier example of a no-bubble theorem of the first category. Santos and Woodford (1997, Theorems 3.1, 3.3) established celebrated no-bubble theorems of this category, which were extended by Huang and Werner (2000, Theorem 6.1) and Werner (2014, Remark 1, Theorem 1) to different settings.

Unlike these results, no-bubble theorems of the second category are mostly based on the optimal behavior of a single agent without relying on the present value of the aggregate endowment process. For example, in a deterministic economy with finitely many agents, Kocherlakota (1992, Proposition 3) showed that asset bubbles can be ruled out if at least one agent can reduce his asset holdings permanently from some period onward.[1] Obstfeld and Rogoff (1986) used a similar idea earlier to rule out deflationary equilibria in a money-in-the-utility-function model.[2] These results rely on the necessity of a transversality condition, and a fairly general no-bubble theorem based on the necessity of a transversality condition was shown by Kamihigashi (2001, p. 1007) for deterministic representative-agent models in continuous time.[3]

In this paper we establish a simple no-bubble theorem of the second category that can be used to rule out asset bubbles in an extremely wide range of deterministic models. We consider the problem of a single agent who faces sequential budget constraints and has strictly monotone preferences. We show that asset bubbles never arise if the agent can reduce his asset holdings permanently

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1 Santos (2006) showed a similar result on the value of money in a general cash-in-advance economy.

2 See Kamihigashi (2008a, b) for results on asset bubbles in related models.

3 See Kamihigashi (2002, 2003, 2005) for further results on necessity of transversality conditions.

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from some period onward. This result is a substantial generalization of Proposition 3 in Kocherlakota (1992). Our contribution is to show that this no-bubble theorem holds under extremely general conditions.

The rest of the paper is organized as follows. In Section 2 we present a single agent's problem along with necessary assumptions, and formally define asset bubbles. In Section 3 we offer several examples satisfying our assumptions. In Section 4 we state our no-bubble theorem and show several consequences. In Section 6 we offer some concluding comments. Longer proofs are relegated to the appendices.

2. The general framework

2.1. Feasibility and optimality

Time is discrete and denoted by $t \in \mathbb{Z}_+$. There is one consumption good and one asset that pays a dividend of d_t units of the consumption good in each period $t \in \mathbb{Z}_+$. Let p_t be the price of the asset in period $t \in \mathbb{Z}_+$. Consider an infinitely lived agent who faces the following constraints:

$$c_t + p_t s_t = y_t + (p_t + d_t)s_{t-1}, \; c_t \geq 0, \quad \forall t \in \mathbb{Z}_+, \tag{2.1}$$

$$s \in \mathcal{S}(s_{-1}, y, p, d), \tag{2.2}$$

where c_t is consumption in period t, $y_t \in \mathbb{R}$ is (net) income in period t, s_t is asset holdings at the end of period t with s_{-1} historically given, and $\mathcal{S}(s_{-1}, y, p, d)$ is a set of sequences in $\mathbb{R}$ with $s = \{s_t\}_{t=0}^\infty$, $y = \{y_t\}_{t=0}^\infty$, $p = \{p_t\}_{t=0}^\infty$, and $d = \{d_t\}_{t=0}^\infty$. We offer several examples of (2.2) in Section 3.1.

Let $\mathcal{C}$ be the set of sequences $\{c_t\}_{t=0}^\infty$ in $\mathbb{R}_+$. For any $c \in \mathcal{C}$, we let $\{c_t\}_{t=0}^\infty$ denote the sequence representation of c, and vice versa. We therefore use c and $\{c_t\}_{t=0}^\infty$ interchangeably; likewise we use s and $\{s_t\}_{t=0}^\infty$ interchangeably, and so on. The inequalities $<$ and $\leq$ on the set of sequences in $\mathbb{R}$ (which includes $\mathcal{C}$) are defined as follows:

$$c \leq c' \quad \Leftrightarrow \quad \forall t \in \mathbb{Z}_t, \; c_t \leq c'_t, \tag{2.3}$$

$$c < c' \quad \Leftrightarrow \quad c \leq c' \text{ and } \exists t \in \mathbb{Z}_+, c_t < c'_t. \tag{2.4}$$

The agent's preferences are represented by a binary relation $\prec$ on $\mathcal{C}$. More concretely, for any $c, c' \in \mathcal{C}$, the agent strictly prefers c' to c if and only if $c \prec c'$. Assumptions 2.1 and 2.2, stated below, are maintained throughout the paper.

Assumption 2.1. $d_t \geq 0$ and $p_t > 0$ for all $t \in \mathbb{Z}_+$.

We say that a pair of sequences $c = \{c_t\}_{t=0}^\infty$ and $s = \{s_t\}_{t=0}^\infty$ in $\mathbb{R}$ is a *plan*; that a plan (c, s) is *feasible* if it satisfies (2.1) and (2.2); and that a feasible plan (c^*, s^*) is *optimal* if there exists no feasible plan (c, s) such that $c^* \prec c$. Whenever we take an optimal plan (c^*, s^*) as given, we assume the following.

Assumption 2.2. For any $c \in C$ with $c^* < c$, we have $c^* \prec c$.

This assumption holds if $\prec$ is strictly monotone in the sense that for any $c, c' \in \mathcal{C}$ with $c < c'$, we have $c \prec c'$. While this latter requirement may seem reasonable, there is an important case in which it is not satisfied; see Example 3.2.

2.2. Asset bubbles

In this subsection we define the fundamental value of the asset and the bubble component of the asset price in period $t \in \mathbb{Z}_+$ using the period t prices of the consumption goods in periods $t, t+1, \ldots$. To be more concrete, let $q_{0,t}$ be the period 0 price of the consumption good in period $t \in \mathbb{Z}_+$. It is well known (e.g. Huang and Werner, 2000), (8)) that the absence of arbitrage implies that there exists a price sequence $\{q_{0,t}\}$ such that

$$\forall t \in \mathbb{Z}_+, \quad q_{0,t}p_t = q_{0,t+1}(p_{t+1} + d_{t+1}), \tag{2.5}$$

$$\forall t \in \mathbb{N}, \quad q_{0,t} > 0, \tag{2.6}$$

$$q_0 = 1. \tag{2.7}$$

Under Assumption 2.1, conditions (2.5) and (2.7) uniquely determine the price sequence $\{q_{0,t}\}$. For the rest of the paper, we consider the price sequence $\{q_{0,t}\}$ given by (2.5) and (2.7).

For $t \in \mathbb{N}$ and $i \in \mathbb{Z}_+$, we define

$$q_{t,t+i} = q_{0,t+i}/q_{0,t}, \tag{2.8}$$

which is the period t price of consumption in period $t+i$. Note that

$$\forall i, j, t \in \mathbb{Z}_+, \quad q_{t,t+i}q_{t+i,t+i+j} = \frac{q_{0,t+i}}{q_{0,t}}\frac{q_{0,t+i+j}}{q_{0,t+i}} = q_{t,t+i+j}. \tag{2.9}$$

Let $t \in \mathbb{Z}_+$. Eqs. (2.5) and (2.8) give us $p_t = q_{t,t+1}(p_{t+1} + d_{t+1})$. By repeatedly applying this equality and (2.9), we obtain

$$p_t = q_{t,t+1}d_{t+1} + q_{t,t+1}p_{t+1} \tag{2.10}$$

$$= q_{t,t+1}d_{t+1} + q_{t,t+1}q_{t+1,t+2}(p_{t+2} + d_{t+2}) \tag{2.11}$$

$$= q_{t,t+1}d_{t+1} + q_{t,t+2}d_{t+2} + q_{t,t+2}p_{t+2} \tag{2.12}$$

$$\vdots \tag{2.13}$$

$$= \sum_{i=1}^{n} q_{t,t+i}d_{t+i} + q_{t,t+n}p_{t+n}, \quad \forall n \in \mathbb{N}. \tag{2.14}$$

Since the above finite sum is increasing in $n \in \mathbb{N}$,[4] it follows that

$$p_t = \sum_{i=1}^{\infty} q_{t,t+i}d_{t+i} + \lim_{n\uparrow\infty} q_{t,t+n}p_{t+n}. \tag{2.15}$$

As is commonly done in the literature, we define the *fundamental value* of the asset in period t as the present discounted value of the dividend stream from period $t+1$ onward:

$$f_t = \sum_{i=1}^{\infty} q_{t,t+i}d_{t+i}. \tag{2.16}$$

The *bubble* component of the asset price in period t is the part of p_t that is not accounted for by the fundamental value:

$$b_t = p_t - f_t. \tag{2.17}$$

It follows from (2.15)–(2.17) that

$$b_t = \lim_{n\uparrow\infty} q_{t,t+n}p_{t+n}. \tag{2.18}$$

Using (2.9) we see that

$$q_{0,t} \lim_{n\uparrow\infty} q_{t,t+n}p_{t+n} = \lim_{n\uparrow\infty} q_{0,t+n}p_{t+n} = \lim_{i\uparrow\infty} q_{0,i}p_i. \tag{2.19}$$

Hence, (2.18) and (2.6) give us

$$\lim_{i\uparrow\infty} q_{0,i}p_i = 0 \quad \Leftrightarrow \quad \forall t \in \mathbb{Z}_+, \; b_t = 0. \tag{2.20}$$

3. Examples

In this section we present several examples of (2.2) as well as examples of preferences that satisfy Assumption 2.2. Some of the examples are used in Section 4.

[4] In this paper, "increasing" means "nondecreasing", and "decreasing" means "nonincreasing".