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Prebidding first-price auctions with and without head starts

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HIGHLIGHTS

- We study first-price auctions where all the bidders' values are private information except bidder 1's value which is commonly known.
- Bidder 1 places his bid before (prebidding auction) all the other bidders.
- We show that regardless of his value, bidder 1 always has a positive effect on the expected highest bid.
- The prebidding first-price auction with n bidders may be less profitable than the optimal simultaneous first-price auction with only $n-1$ bidders.
- The prebidding first-price auction with the optimal head start is always more profitable than the optimal first-price auction with $n-1$ bidders.

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ABSTRACT

We study the effect of prebidding in first-price auctions with a single prize under incomplete information. The values of the $n-1$ bidders are private information while bidder 1's value is commonly known. Bidder 1 places his bid before all the $n-1$ bidders. We show that regardless of his value, bidder 1 always has a positive effect on the expected highest bid. However, bidder 1's contribution to the expected highest bid is not significant since the prebidding first-price auction with n bidders may be less profitable than the optimal simultaneous first-price auction with only $n-1$ bidders. On the other hand, by giving the optimal head start to bidder 1, the prebidding first-price auction is always more profitable than the optimal simultaneous first-price auction with $n-1$ bidders

1. Introduction

One of the most fundamental results in auction theory is the revenue equivalence theorem established by Myerson (1981) according to which the expected revenue of the seller in equilibrium is independent of the auction mechanism under quite general conditions.[1] One of these conditions is that the bidders are ex-ante symmetric. However, when the symmetry of the bidders is broken the revenue equivalence of auction mechanisms does not hold.[2] The symmetry of the bidders is broken, for example, when the bidders submit their bids sequentially rather than simultaneously. Bids are submitted sequentially particularly when the seller wishes to favor specific bidders (Laffont and Tirole, 1991). Favoritism of one (or several) of the bidders can occur when the favorite bidder obtains the option to observe the bids of all the other bidders and only then submits his bid (Arozamena and Weinschelbaum, 2009), or, alternatively, when the seller offers the option for one of the bidders to change his bid for a bribe (Menezes and Monterio, 2006). Favoritism can also occur by letting a bidder submit his bid before all the others and then giving him a head start.

To illustrate a sequential auction, consider a firm that won the exclusive license to operate during some period of time. When this period is over, the operating licence is resold by an auction. The candidates include the firm that won it in the previous period and several new competitors who covet it. The difference between them is that the information about the firm is commonly known, while the information about the new competitors is private information. Now suppose that the auctioneer prefers that the firm will win again since the firm's abilities to use and take advantage of the exclusive license has already been proved while the implications of another winner are uncertain. In that case, the firm can be given the right to bid with or without a head start before the other bidders who have observed the firm's bid before they place their own bids.

Formally, we study sequential first-price auctions under incomplete information where bidder 1's value for the object is

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1 See Lorentziadis (2016) for more information on optimal bidding in auctions from a game theory perspective.

2 For revenue equivalence of asymmetric auctions, see, for example, Fibich and Gavious (2003) and Fibich et al. (2004).

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commonly known while the other $n-1$ bidders' values are private information.[3] Our aim is not to analyze the optimal auction mechanism. Instead, it is to analyze how the seller maximizes his expected payoff in the above sequential first-price auction when he gives bidder 1 some advantage by enabling him to place his bid before the other bidders with (or without) a head start.

We analyze this prebidding first-price auction with n bidders and show that although bidder 1's bid plays the role of a reserve prize for all the other bidders in the second stage, regardless of his value for the object, he has a positive effect on the expected highest bid. Furthermore, the expected highest bid increases in bidder 1's value for the prize. The intuition for these results is that, in contrast to the standard simultaneous auction, the expected highest bid in the prebidding auction is larger than or equal to the reserve price which is bidder 1's bid in the first stage. Nonetheless, we show that the contribution of bidder 1 to the expected highest bid is not always significant. In particular, we show that for sufficiently low values of bidder 1, the expected highest bid in the simultaneous first-price auction with $n-1$ bidders and a reserve price that equals bidder 1's value is higher than in the prebidding auction with n bidders. This result shows that the well-known result of Bulow and Klemperer (1996) according to which a private-value ascending auction with no reserve price and n symmetric bidders is more profitable than any auction with $n-1$ of these bidders does not hold in our environment when we let the bidders to submit their bids sequentially.[4] However, by allowing the seller to give a head-start to bidder 1 such that each of the bidders in the prebidding auction wins if his bid is larger than or equal to kb_1 where b_1 is bidder 1's bid in the first stage and k is a constant larger than one, then the prebidding first-price auction with n bidders is more profitable than the optimal first-price auction with $n-1$ bidders. Thus, by allowing head starts we demonstrate the robustness of Bulow and Klemperer's result in our environment.

Several papers in the literature study head starts in contests. Corns and Schotter (1999) demonstrated by theoretical and empirical arguments that a head start in the form of a price preference policy that is given to a subset of firms might not only benefit that subset but can actually lower the purchasing cost of the government. Kirkegaard (2012) studied asymmetric all-pay auctions with head starts under incomplete information when bidders simultaneously choose their efforts, and showed that the total effort increases if the weak contestant is favored with a head start. Segev and Sela (2014) revealed that in a sequential all-pay auction by giving a head-start to the bidder who places the first bid a designer can increase the bidders' expected highest bid.

Sequential auctions have also received some attention. For example, Pitchik and Schotter (1988) analyzed sequential first and second price auctions with a budget constraint and two different objects. Benoit and Krishna (2001) analyzed sequential first and second price auctions with synergy between the stages and a budget constraint, and Pitchik (2009) analyzed a sequential auction with a budget constraint under incomplete information.[5] All these papers deal with sequential auctions in which an object is awarded in each stage of the auction when the link between the stages is made by the bidders' budget constraints. In contrast, in our paper only one object is awarded in the last stage which links between the stages of the auction.

The rest of the paper is organized as follows: In Sections 2 and 3 we analyze our prebidding auctions and compare between the sequential first-price auction with n bidders and the simultaneous first-price auction with $n-1$ bidders with and without head starts. In Section 4 we conclude.

2. The prebidding first-price auction

We begin with a first-price auction with n risk neutral bidders and an indivisible object where bidder 1 has a commonly known value for the object v_1, while bidder $i, i = 2, 3, \ldots, n$ has a private value for the object v_i which is drawn independently from a continuously differentiable distribution function $F(v)$ over the support $[0, 1]$ with a positive and continuous density function $f > 0$.[6] In the first stage bidder 1 submits a bid $b_1(v_1)$, and in the second stage, the other $n-1$ bidders observe bidder 1's bid and then each of them simultaneously submits a bid $b_i(v_i), i = 2, 3, \ldots, n$. The bidder with the highest bid wins the object and pays his bid. In the case of a tie, one of the $n-1$ bidders who placed the highest bid in the second stage wins. We term the above form of a sequential first-price auction a prebidding first-price auction.

In order to analyze the perfect Bayesian equilibrium of this sequential auction we begin with the second stage and go backwards to the previous one. Assume that in the second stage there is a monotonic and differentiable equilibrium bid function $x_i = b_i(v_i), i = 2, \ldots, n$. The inverse bid function in this case will be defined as $y_i(x_i)$. Then, the maximization problem of bidder i is

$$\max_x U_i(x) = F^{n-2}(y_i(x))(v_i - x)$$
$$s.t. \quad x \geq b_1$$

where $b_1 = b_1(v_1)$ is bidder 1's bid in the first stage. The solution of the above maximization problem yields the equilibrium strategies of the standard (simultaneous) first-price auction with $n-1$ symmetric bidders who face a reserve price of $b_1 = b_1(v_1)$. That is, for every $i = 2, \ldots, n$,

$$b(v_i) = \begin{cases} 0 & \text{if} \quad 0 \leq v_i < b_1 \\ v_i - \dfrac{1}{F^{n-2}(v_i)} \displaystyle\int_{b_1}^{v_i} F^{n-2}(s)ds & \text{if} \quad b_1 \leq v_i \leq 1. \end{cases} \tag{1}$$

Then, the maximization problem of bidder 1 in the first stage is

$$\max_x U_1(x) = F^{n-1}(x)(v_1 - x).$$

The F.O.C. is

$$\frac{\partial U_1(x)}{\partial x} = (n-1)F^{n-2}(x)f(x)(v_1 - x) - F^{n-1}(x) = 0.$$

By substituting $y(x) = v$, $x = b_1(v)$ and rearranging, we obtain that the equilibrium strategy of bidder 1 is

$$\frac{F(b_1(v))}{(n-1)f(b_1(v))} = v - b_1(v) \tag{2}$$

or alternatively,

$$b_1(v) = v - \frac{F(b_1(v))}{(n-1)f(b_1(v))}.$$

Differentiating with respect to v we get

$$b_1'(v) = 1 - b_1'(v)\left(\frac{f^2(b_1(v)) - F(b_1(v))f'(b_1(v))}{(n-1)f^2(b_1(v))}\right).$$

A rearrangement yields

$$b_1'(v) = \frac{1}{1 + \frac{f^2(b_1(v)) - F(b_1(v))f'(b_1(v))}{(n-1)f^2(b_1(v))}}. \tag{3}$$

[3] On asymmetric first-price auctions under incomplete information, see, for example, Lebrun (1999), Kirkegaard (2009) and Kaplan and Zamir (2012).

[4] Bulow and Klemperer show that this result holds also for a wide class of common-value auctions.

[5] See also Brusco and Lopomo (2008, 2009) who considered sequential auctions with a budget constraint and with and without synergy between the values of the objects.

[6] In the following we sometimes assume that the distribution function F is decreasing reversed hazard rate (DRHR). Increasing hazard rate (IHR) distributions like Weibull, gamma and lognormal distributions were found to be DRHR. In addition, a decreasing failure rate (DHR) distribution is necessarily a DRHR distribution (see Barlow and Proschan, 1966, 1975).

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