Mathematical Social Sciences 73 (2015) 50–54

Contents lists available at ScienceDirect

Mathematical Social Sciences

journal homepage: www.elsevier.com/locate/econbase

A class of symmetric and quadratic utility functions generating Giffen demand☆

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HIGHLIGHTS

- I provide an example of utility function that generates a Giffen demand.
- Although piecewise defined, this utility is quite simple: symmetric and quadratic.
- This utility represents standard preferences in Economics.
- I characterize the parameter values under which Giffen demand is obtained.

ARTICLE INFO

Article history:
Received 1 June 2012
Received in revised form
12 November 2014
Accepted 21 November 2014
Available online 5 December 2014

ABSTRACT

I provide a simple example of a quadratic utility function that generates a Giffen demand. The utility function is symmetric, increasing and concave. Interestingly, the Giffen effect arises in the subspace where the utility function is strictly increasing and strictly concave. A full characterization of the parameter conditions under which the Giffen demand arises is provided.

1. Introduction

The existence of Giffen goods beyond the theoretical possibility that was then formalized in the Slutsky equation, has been questioned for a long time (see, for example, Vandermeulen, 1972; Stigler, 1947). Recently, Jensen and Miller (2008), have found evidence of Giffen behavior for consumption of rice in rural China.[1] This discovery brings back to relevance the understanding of the micro foundations of such a demand function.[2]

This note provides an example of a simple, smooth and conventional utility function that generates a Giffen demand.

Although the utility function is piecewise, it is conventional in the sense that it represents monotone and convex preferences generating a continuous demand function. Interestingly, the Giffen good property is observed in the subset of the positive quadrant where preferences for both goods are strictly monotone and strictly convex.

Earlier examples relied on non-conventional utility functions (Spiegel, 1994; Weber, 1997; Vandermeulen, 1972). Nonconvex preferences also appear in more recent works (Butler and Moffatt, 2000; Heijman and Mouche, 2012). Starting from the price offer curve Moffatt (2002) constructs a family of conventional indifference curves (hyperboles) that generate the Giffen property, without providing any specific utility function[3]; Sørensen (2007) modifies Leontief utility functions by introducing some degree of substitutability between the two commodities but does not obtain smooth indifference curves. Doi et al. (2009) provide an example of a piecewise monotone and convex utility function that generates a

☆ I would like to thank the Editor and two referees for their comments that have improved the paper substantially. Special thanks also go to Domenico Menicucci for his helpful comments and discussions, and the Department of Mathematics for Decisions at the University of Florence for their hospitality. All the remaining mistakes are mine.

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1 In particular, they provide evidence that demand for rice has the Giffen property amongst poor people who spend a quite significantly large share of their income in it and for which there are some substitutes who are more expensive.

2 The recent works by Doi et al. (2009) and Heijman and Mouche (2012) provide an exhaustive and well crafted introduction to this topic.

3 Heijman and Mouche (2012) and Moffatt (2012) use Moffatt (2002) approach to provide some examples, which are not that simple nor conventional.

http://dx.doi.org/10.1016/j.mathsocsci.2014.11.006

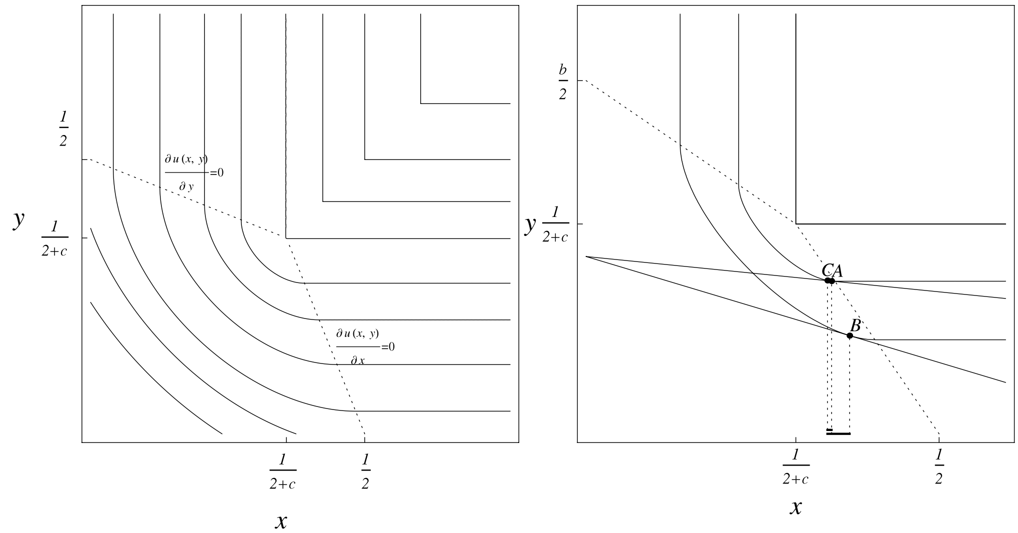

Fig. 1. Left: Indifference maps. Right: Income ($\overline{CB}$) and Substitution ($\overline{AC}$) effects.

Giffen demand. This note fits into the same category, and provides an even simpler utility function.

2. The family of utility functions

Consider a utility function whose indifference curves are reported in the left panel of Fig. 1. Preferences are monotone and convex in $\mathbb{R}^2_+$. The dotted lines represent the reaction functions $\partial u(x, y)/\partial x = 0$ and $\partial u(x, y)/\partial y = 0$. Utility is strictly increasing and strictly concave in the subset of $\mathbb{R}^2_+$ delimited by these dotted lines. The Giffen effect arises exactly in this region, as shown in Fig. 2.

The top panel shows the price consumption path obtained by changing p. The bottom panel indicates the demand for x so derived. We can see that demand is upward sloping and convex for a nonempty and convex subset of the prices. At the current income, the solution to the utility maximization problem is never at a kink. Therefore demand for x can be constructed via simple tangency considerations. The right panel of Fig. 1 shows the income and substitution effects associated to a certain price increase of x.

This example is generated from the following utility function

$$w(x, y) = -a(x^2 + y^2) + b(x + y) - cxy + d. \tag{1}$$

There is no loss in generality in setting $a = 1$ and $d = 0$. Assume that $b > 0$ and $2 > c > 0$. This utility function represents preferences that are strictly convex and with a satiation point at $(b/(2 + c), b/(2 + c))$. We can construct a monotone and concave utility function (i.e. one representing standard preferences) as follows.

$$u(x, y) = \begin{cases} w(x, y) \quad \text{if } 2x + cy > b \text{ and } cx + 2y > b \\ \frac{1}{4}\left(b^2 - 2b(c-2)x + \left(c^2 - 4\right)x^2\right) \\ \quad \text{if } 0 \le x \le \frac{b}{c+2} \text{ and } cx + 2y < b \\ \frac{1}{4}(b^2 - 2b(c-2)y + (c^2 - 4)y^2) \\ \quad \text{if } 0 \le y \le \frac{b}{c+2} \text{ and } 2x + cy > b \\ \frac{b(b-1)}{2+c} + \min\{x, y\} \\ \quad \text{if } x \ge \frac{b}{2+c} \text{ and } y \ge \frac{b}{2+c}. \end{cases} \tag{2}$$

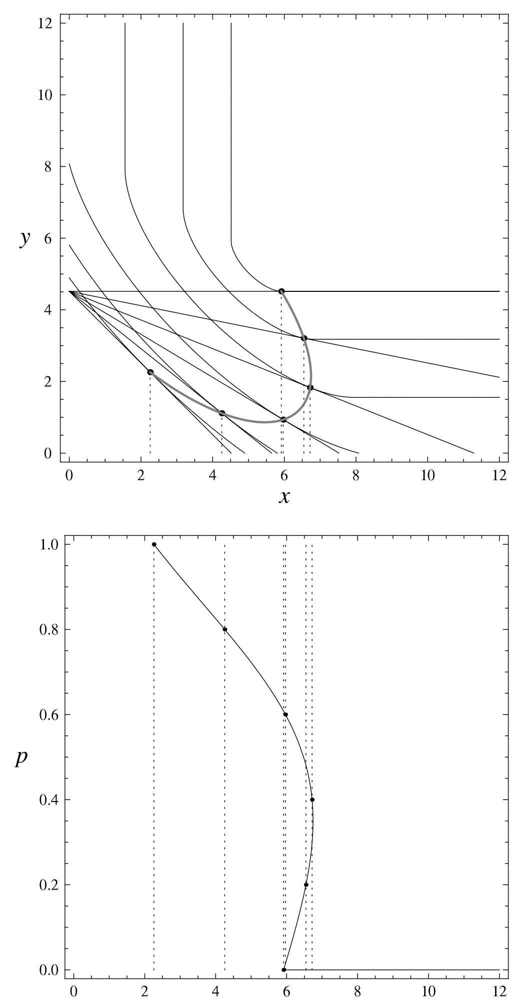

Fig. 2. Top: Income consumption path as price of x changes. Bottom: Demand curve for x which shows the Giffen property.

The utility is constructed by creating flat (horizontal or vertical) indifference curves for the good whose marginal utility has become negative according to $w(x, y)$. With quadratic utility, we cannot exclude corner solutions a priori. This complicates a bit the