North American Journal of Economics and Finance xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

North American Journal of Economics and Finance

journal homepage: www.elsevier.com/locate/najef

Compound option pricing under a double exponential Jump-diffusion model

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ARTICLE INFO

Keywords:
Compound option
Double exponential distribution
Jump-diffusion process

ABSTRACT

A compound option, an option on another option, plays an important role in financial field since it can be used to price American option and corporate debt with discrete coupons. In the real options literature, compound options are most suitable to be employed to investment problems involving sequential decision making. Most compound option and real options formulae are based on log-normal distribution while the empirical evidence shows that the return distribution in real market exhibits asymmetric leptokurtic feature, higher peak and two heavier tails. This paper introduces the jump-diffusion process into pricing compound options and derives the related valuation formulas. We assume that the dynamic of the underlying asset return process consists of a drift component, a continuous Wiener process and discontinuous jump-diffusion processes which have jump times that follow the compound Poisson process and the logarithm of jump size follows the double exponential distribution proposed by Kou (2002). Numerical results indicate that the advantage of combining the double exponential distribution and normal distribution is that it can capture the phenomena of both the asymmetric leptokurtic features and the volatility smile. Furthermore, the compound options under the double exponential jump diffusion model which we derived are more generalized than those proposed by Gukhal (2004) and Geske (1979), and thus have wider application.

1. Introduction

Compound options have been extensively used in valuing financial applications. It might be thought of as simply an option on an option by Black and Scholes (1973) (hereafter, B-S (1973)). That is to say, the underlying asset of a compound option can be any form of option. When exercising a compound option, the option holder will buy or sell another option on an equivalent basis. Essentially, according to the underlying assets, such as calls or puts, compound options can be divided into four categories: (1) call on call; (2) call on put; (3) put on call; and (4) put on put.

A compound option plays an important role in financial field since it is a multifunctional and powerful tool for resolving a lot of financial problems. It can be used to price American-style option and corporate debt with discrete coupons. Geske and Johnson (1984a) used exotic multi-fold compound options for American put options, while Carr (1988) presented the pricing formula for compound exchange options by integrating the exchange option pricing of Fischer (1978) and Margrabe (1978) into the compound option pricing of Geske (1979). Geske and Johnson (1984b) and Rubinstein (1992) used compound option approaches to value corporate debt and chooser options, respectively. Options on interest rate options, like captions and floortions derived by Musiela and

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https://doi.org/10.1016/j.najef.2017.10.002
Received 21 November 2016; Received in revised form 29 September 2017; Accepted 3 October 2017

Rutkowski (1998), are also priced by compound options. In addition to the pricing of financial derivatives, compound option theory is widely applied in real option studies. To evaluating investment problems involving sequential decision making in the real options literature is always relied on compound options. Myers (1977) first presented this approach, and the ideas were developed by Brennan and Schwartz (1985), Pindyck (1988), and Trigeorgis (1993), Trigeorgis (1996), among other works. Examples applications of this approach include project valuations of new drugs (Casimon, Engelen, Thomassen, & Van Wouwe, 2004), production and inventory (Cortazar & Schwartz, 1993) and capital budget decisions (Duan, Lin, & Lee, 2003). Compound option methodology is thus very widely applied, and the theory is versatile enough to deal with many real-world cases (Copeland & Antikarov, 2003). Take Casimon et al. (2004) for example, the real option valuation via a compound option approach to new drug applications (NDA) and the R&D of pharmaceutical companies. The development of an NDA starts with a pre-clinical test phase, followed by three clinical test phases, FDA approval, and ultimately commercialization. If the initial R&D turns out to be successful, the pre-clinical phase is started, otherwise the research is discontinued. Therefore, the process can be viewed as a twofold compound option. Another prominent application of the compound option approach is insurance policies with sequential premiums, and it is also used with installment options. Worldwide, the most actively traded installment options are currently the installment warrants on Australian stocks listed on the Australian Stock Exchange (ASX). In general, at each decision date, the installment option holder has three choices to follow: First, to exercise the option. Second, not to exercise the option and to pay the installment to keep the option alive until the next decision date. Third, not to exercise the option and also not to pay the installment, and thus to let the contract expire. In other words, the installment option holders have the right to decide periodically whether to exercise or not, and then to keep the option alive or not by paying the installment at each decision date. Moreover, the two steps of a installment option can be viewed as a compound option. There is also some research available on the valuation of installment options, such as Davis, Schachermayer, and Tompkins (2001), Ciurlia and Roko (2005) and Ben-Ameur, Breton, and Francois (2006).

Most compound option and real options formulae are based on log-normal distribution while the empirical evidence shows that the return distribution in real market exhibits asymmetric leptokurtic feature, higher peak and two heavier tails. Casimon et al. (2004) point out that most real option valuation problems are based on Geske's model, which means that the present value of the free operating cash flows (or stock price) does not consider the economic conditions of either a volatility smile or non-normal distribution for asset returns. Generally speaking, there are two phenomena investigated by many of the related empirical studies: (1) the asymmetric leptokurtic features, and (2) the volatility smile. In B-S (1973), the option pricing framework assumes that the underlying assets follow a log-normal distribution with a constant volatility during the whole life of an option, with various strike prices and maturities. However, the reality is that the return distribution is skewed to the left and has a higher peak and two heavier tails than the normal distribution (Fama, 1965; Lo, 1991; Lo & MacKinlay, 1988). Second, the implied volatilities in B-S (1973) either decrease with the exercise price, increase with the exercise price, or have a U-shaped pattern across exercise prices, which is known as the volatility smirk. Moreover, many empirical studies indicate that the implied volatilities are higher for short-term options than they are for long-term options. In other words, since all the pricing models based on B-S (1973)'s model assume constant implied volatilities exist across different strike prices and maturities, they cannot explain these empirical regularities.

In light of the fact outlined above, there have been a considerable number of studies that attempt to revise the B-S (1973) model. Merton (1976) derived the closed-form solutions for European options using a jump-diffusion process. The jump component in this model is compound Poisson process and jump amplitude is drawn from a lognormal distribution. Sundaresan (2002) indicated that while geometric Brownian motions had served as a convenient and tractable framework, but the main reason that Merton (1976) gained wide acceptance was because it was being more consistent with the empirical return distributions. Kou (2002) proposed a unique jump diffusion model by using the double exponential distribution. It has desirable properties for both pricing exotic options and econometric estimation. This model can not only overcome the two empirical puzzles, the leptokurtic and asymmetric features and the volatility smile, but also leads to analytical solutions to many option pricing problems, including vanilla options and interest rate derivatives, even for exotic options, such as perpetual American options, barrier, lookback options (Kou and Wang (2004)) and catastrophe options (Jiang, Yang, Liu, & Wang, 2013). Sepp (2004) compared jump diffusions with double exponential jumps, negative exponential jumps, and normally distributed jumps in order to discover which can best be calibrated to the market. Sepp demonstrated that the obvious advantage of the double exponential jump diffusion is that it exhibits higher peaks, which are inherent to the distributions of the asset returns. Ramezani and Zeng (2007) assessed the performance of the double exponential jump-diffusion model relative to the log-normally distributed jump-diffusion model and the geometric Brownian motion model. They found that both the models of Kou (2002) and Merton (1976) are superior to that of B-S (1973). Furthermore, for majority of the individual stocks and the indexes, the Kou's model provides a better fit than Merton's.

Since Geske (1979) derived a successful analytical solution for compound options when the firm value follows a geometric Brownian motion, it has turned out that a wide variety of important problems are closely related to the valuation of compound options. Most of related research focuses on American option pricing, and to date there has been relatively little research into incorporating non-normality or volatility smiles to make the option pricing model more accurate. Gukhal (2004) applied the compound options approach to American options when the asset price evolves as a jump-diffusion process and pays either discrete or continuous dividends, and derived analytical solutions for compound and extendible options. However, the jump-diffusion process employed by Gukhal (2004) is the compound Poisson process first proposed by Merton (1976), and this model cannot solve the two empirical puzzles. For this purpose, this paper develops a generalize compound option pricing model with jump diffusion process in which the jump amplitude of underlying return following a double exponential distribution instead of normal distribution. The compound options are widely applied in financial field. For example, they can be used to discuss the evaluation of American style option and the corporate bond with discrete coupons (Roll, 1977; Geske, 1979; Whaley, 1981; Geske & Johnson, 1984; Longstaff, 1990). They can also be used to resolve the sequential investment decisions in real options field (Kemna, 1993; Keeley, Punjabi, &