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Estimation of multifractality based on natural time analysis

A. Mintzelas [a], N.V. Sarlis [a,b,*], S.-R.G. Christopoulos [c,b]

[a] *Section of Solid State Physics, Department of Physics, School of Science, National and Kapodistrian University of Athens, Panepistimiopolis, Zografos 157 84, Athens, Greece*
[b] *Solid Earth Physics Institute, Department of Physics, School of Science, National and Kapodistrian University of Athens, Panepistimiopolis, Zografos 157 84, Athens, Greece*
[c] *Faculty of Engineering, Environment and Computing, Coventry University, Priory Street, Coventry CV1 5FB, United Kingdom*

HIGHLIGHTS

- A method to study multifractals based on natural time is presented.
- Generalized fluctuations of natural time is the key quantity of this method.
- Comparison is made with well established methods like MFDFA and MFCMA.
- The new method is applied to heart rate variability with interesting results.

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ABSTRACT

Recent studies have shown that results deduced on the basis of a new time domain termed natural time reveal that novel dynamical features hidden behind time-series in complex systems can be uncovered. Here, we propose a method for estimating the multifractal behavior of time series by studying the fluctuations of natural time under time reversal. Examples of the application of this method to fractional Gaussian noises, fractional Brownian motions, binomial multifractal series, Lévy processes as well as interbeat intervals' time series from electrocardiograms are presented.

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1. Introduction

In most cases, the time-series resulting from physical systems and processes exhibit correlations that decay exponentially. However, it is well known [1,2] that a major exception is when approaching a critical point, the exponential decay turns into a power-law decay. Moreover, long range power-law correlations have been found in a wide variety of systems including complex systems, e.g., see Ref. [3]. Such systems give rise to time-series that exhibit scale-invariant features characterized by long-range power law correlations. Since their superposition with erratic fluctuations due to, for example, noise in the emitted signals, may be unavoidable or the amount of experimental data may be small, it is very important to investigate techniques that may identify such long-range power law correlations. This is main the scope of the present paper by employing the new time domain termed natural time [4–8] which may uncover hidden properties in the time-series of complex systems.

* Corresponding author at: Section of Solid State Physics, Department of Physics, School of Science, National and Kapodistrian University of Athens, Panepistimiopolis, Zografos 157 84, Athens, Greece.
E-mail addresses: kypheidon@phys.uoa.gr (A. Mintzelas), nsarlis@phys.uoa.gr (N.V. Sarlis), ac0966@coventry.ac.uk (S.-R.G. Christopoulos).

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In general, a stochastic process $X(t)$ is called [9,10] self-similar with index H if it has the property $X(\lambda t) \stackrel{d}{=} \lambda^H X(t)$, where $\stackrel{d}{=}$ denotes the usual equality of finite-dimensional distributions. Such a process generates time series which is characterized by an unique scaling exponent H in their entire length. This kind of time series is called monofractal time series [11].

In reality, the majority of time series does not exhibit a simple monofractal scaling behavior, which can be accounted for by a single scaling exponent H. As mentioned in Ref. [12], there is the possibility of crossover (time-) scales s_x separating regimes with different scaling exponents [13,14], e.g. long-range correlations on small scales s ($s \ll s_x$) and another type of correlations or uncorrelated behavior on larger scales s ($s \gg s_x$). In other cases, the scaling behavior is more complicated, and different scaling exponents are required for different parts of the series [15]. Such time series, where more than one scaling exponent (set of scaling exponents) is needed in order to describe their scaling properties are called multifractal time series, e.g., [16–25].

Detrended Fluctuation Analysis (DFA) constitutes a well-established method for the determination of the scaling exponent H in monofractal time series [26,27] as well as its generalization Multifractal Detrended Fluctuation Analysis (MFDFA) in the case of multifractal time series [12]. Another widely adopted [28] method for estimating the multifractal behavior in non-stationary observational records, is the wavelet transform modulus maxima (WTMM) method [29,30] which involves, however, a more complicated mathematical treatment. WTMM has been compared [12,31,32] to MFDFA and the results show [33] that MFDFA is at least equivalent to WTMM, while the latter needs more care and may yield spurious multifractality [32]. An alternative multifractal method that has [33] even less computational difficulties than MFDFA, is the one based on the Centered Moving Average (CMA) technique [34,35], that improves the classical causal backward moving average method [36,37] (see also [38]), and is called multifractal CMA (MFCMA). Since CMA performs better than DFA in the limit of very small and very large scales, MFCMA is more suitable [33] for short time series. Of course, the detrending made in MFCMA is not as strong as in MFDFA, thus one may consider MFCMA for data without significant trends and MFDFA for data with intense trends [33], like polynomials of large curvature or periodicities of high amplitude and/or frequency (for the importance of proper detrending see also [39]). Natural time [4–8] and more specifically the fluctuations of its average value under time reversal, can also capture [40] the scaling properties of a monofractal time series by means of the determination of its scaling exponent H. Here, we attempt a generalization of this method to the multifractal case.

In Section 2, we describe the basics of natural time analysis including the fluctuations of the average value of natural time under time reversal and introduce the Generalized Fluctuations of the average value of Natural Time under time reversal (GFNT). In the same Section, we present the methodology we adopt to estimate the set of generalized scaling exponent $h(q)$ of a multifractal time series and its singularity spectrum $f(\alpha)$. Furthermore, in Section 3, we examine various time series, comparing the results coming from the different methods: MFDFA, MFCMA and GFNT. Finally, Section 4 summarizes our conclusions.

2. Methodology

2.1. Natural time analysis

In a time series comprising N events, the natural time $\chi_k = k/N$ serves as an index for the occurrence of the kth event [4,5]. In natural time analysis the evolution of the pair (χ_k, Q_k) is considered, where Q_k denotes in general a quantity proportional to the energy released in the kth event. For example, for dichotomous signals, Q_k stands for the duration of the kth pulse while for the seismicity Q_k is proportional to the seismic energy released during kth earthquake [6,41–48]. Usually instead of Q_k, the normalized energy release $p_k = Q_k/\sum_{i=1}^{N} Q_i$ is used [7]. The latter sum up to unity, $\sum_{k=1}^{N} p_k = 1$, and can be considered as probability [49] giving rise to an average value of natural time $\langle\chi\rangle = \sum_{k=1}^{N} \chi_k p_k$. This value changes [40] if we consider the action [50–52] of the time-reversal $\hat{T} p_k = p_{N-k+1}$ that makes the first pulse to be considered as the last one, the second as the last but one etc.

It is noteworthy that the physical meaning of the difference of average value of natural time χ under time reversal, i.e., of the quantity $\langle\chi\rangle - \langle\hat{T}\chi\rangle$, can be revealed if we consider the parametric family of the distributions $p(\chi;\epsilon) = 1 + \epsilon(\chi - 1/2)$. For small ϵ ($\ll 1/2$), these distributions correspond to small trends superimposed on a uniform distribution [22,53]. Direct calculation of $\langle\chi\rangle \equiv \int_0^1 \chi p(\chi;\epsilon) d\chi$ yields $\langle\chi\rangle = 1/2 + \epsilon/12$ and since $\hat{T} p(\chi;\epsilon) = p(\chi;-\epsilon)$, one obtains that $\langle\chi\rangle - \langle\hat{T}\chi\rangle = \epsilon/6$. Thus, the average value of natural time under time reversal is proportional to the 'local' trend ϵ and the study of its fluctuations, which is the subject of the next subsection, is actually the study of how these 'local' trends fluctuate in a time-series.

2.1.1. Fluctuations of the average value of natural time under time reversal

In order to study the long-range dependence in a time series, e.g., Q_k, $\{k = 1, 2, \ldots, N\}$, we have to define a scale-dependent measure (for example, the detrended fluctuation $F_d(l)$ constitutes [26] such a measure in DFA). Natural time and particularly the fluctuations of natural time under time reversal, may constitute [40] such a scale-dependent measure enabling us to introduce a reliable method to extract the scaling exponent H of a monofractal time series.

The fluctuation of the average value of natural time under time reversal within scale l is

$$\Delta\chi_l = \sqrt{E[(\langle\chi\rangle - \langle\widehat{T}\chi\rangle)^2]} \quad (1)$$

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