Physica A 512 (2018) 174–185

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Feedback-based control for coupled map car-following model with time delays on basis of linear discrete-time system

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HIGHLIGHTS

- Feedback control of coupled map car-following model with times delay is proposed.
- The sufficient condition of stability for the proposed model is given.
- The feedback-based control strategy is designed.
- The effectiveness of the proposed method is verified by numerical simulation.

ARTICLE INFO

Article history:
Received 24 November 2017
Received in revised form 14 May 2018
Available online xxxx

Keywords:
Traffic flow
Feedback-based control
Discrete-time system
Time delays
Traffic congestion

ABSTRACT

In order to suppress traffic jams and make traffic flow smooth, we propose feedback-based control strategy for the coupled map car-following model with time delays on basis of linear discrete-time system. In this new kind of model, time delay is incorporated into dynamic equation of vehicles. The traffic system is analyzed based on Gerschgorin disk theorem and general Nyquist stability criterion. The sufficient condition for the existence of coupled map car-following model with time delays is given. Considering time delays, the proposed control scheme can be competent to alleviate traffic congestion. Numerical examples are given to illustrate the effectiveness of the proposed control method.

1. Introduction

In recent decades, the dynamics of traffic systems have been widely investigated. In order to reveal the mechanisms and characteristics of the traffic phenomena, numerous mathematical traffic models are developed, including such as car-following models [1–15], lattice hydrodynamic models [16–21] and macroscopic traffic flow model [22–26]. Car-following models could describe traffic flow dynamics from microscopic perspective, which reveals some important characteristics of traffic jam phenomena, e.g. stop-and-go wave and phantom jams. So, car-following models attracted wide attention from many investigators and engineers. Stability is one of key elements for the characteristics of traffic flow, which focuses on dynamic behaviors and evolutions under perturbations [1–5,8]. Instabilities of traffic flow can cause frequent

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https://doi.org/10.1016/j.physa.2018.08.122
0378-4371/

acceleration and deceleration, even result in traffic waves. It is important to reveal that there exist various complex instability mechanisms in traffic flow.

Instabilities of traffic flow are investigated extensively by mathematicians and physicists [27–32]. Optimal velocity (OV) model proposed by Bando et al. [1] had attracted great attention. A coupled map OV model with decentralized delay feedback control strategy was proposed [33] for suppressing traffic congestion. Based on coupled map OV model, Zhao et al. [34] presented a simple control method of coupled map car-following model under open boundary, which considered relative velocity of preceding vehicle and host vehicle. Considering intelligent transportation systems (ITS), a modified coupled map car-following model was proposed by Han et al. [35]. The control signal was proportional to the velocity difference between of multiple preceding vehicles. In addition, some modified coupled map OV models were proposed to direct at different conditions, such as road structures [36,37], multiple vehicles systems [38,39] and drivers' behavior [40,41].

Time delays are an important and crucial reason for instability of traffic flow which attracts the wide attention of the researchers [42–45]. Delays in car-following models have been investigated extensively since 1961year. OV model with time delays was proposed by Bando et al., which rediscovered some interesting dynamical behavior [42]. Car-following model considering driver's reaction time was put forward to reveal the oscillations in vehicle velocity induced by encountering slower vehicles [43]. The local and global bifurcations of car-following model with delays are investigated and different periodic bifurcations of traffic state were analyzed [44,45]. Some researchers focused on delays in sensing headway and velocity and then analyzed the impact of traffic congestion [46]. Some works tried to understand comprehensive influence of multi-anticipative driving behavior and/or time delays on traffic flow dynamics and proposed car-following models with different time delays [47–49]. Those contributions indicated that delays affect the evolution of traffic flow from different perspectives.

The present existing literatures mainly studied the dynamics mechanism of traffic flow for continuous car-follow model. Some works also focused on control strategies for suppressing traffic jams and reducing energy consumption, such as sliding mode control [50,51], rolling horizon control [52] and feedback control [53]. However, they drew little attention to address the discrete-time model with time delays about traffic flow. In practice, most control systems of traffic dynamics are implemented on digital hardware. Typically, the controllers designed in the continuous-time domain are individually converted to digital form at the end of the design phase using standard discretization techniques [54,55]. This process is legitimate when the designer can implement the controllers with the sampling and update rates of their choice. Furthermore, the discrete-time controllers provide satisfactory performances in case the computing capabilities are sufficient. In this paper, we propose the feedback control of coupled map car-following model with time delays on the basis on linear discrete-time system. We adopt Nyquist stability criterion and Gerschgorin disk theorem to investigate the sufficient conditions for the stability of coupled map car-following system with time delays. Moreover, different time delays for affecting traffic flow stability are discussed. Finally, the control signal is designed to suppress traffic jams and numerical simulations are shown.

The rest parts of this paper are organized as follows: In Section 2, the CM car-following model with time delays is introduced and its stability analysis is derived. In Section 3, feedback-based control scheme is designed to suppress traffic congestion. In Section 4, numerical simulations are shown. In Section 5, conclusions are summarized.

2. CM car-following model with time delays and its stability analysis

2.1. CM car-following model with time delays

We consider the initial CM car-following model showed in [33,34] under the open boundary condition. The leading vehicle is described as follows:

$$x_l(n+1) = v_0 T + x_l(n) \tag{1}$$

where $x_l(n) > 0$ is the position of the leading vehicle at time $t = nT$. v_0 is its speed of leading vehicle and $T > 0$ is the sampling time. We assume that the lead vehicle is not influenced by others.

The following vehicles are given as:

$$x_i(n+1) = v_i(n)T + x_i(n), (i = 1 \sim N) \tag{2}$$

where $x_i(n) > 0$ is the position of the ith vehicle at time $t = nT$, $v_i(n) > 0$ is the ith vehicle speed and N is the number of the following vehicles. The speed of the following vehicles is governed:

$$v_i(n+1) = \alpha_i \left[V_i^{op}\left(y_i(n-\tau_i)\right) - v_i(n-\tau_i)\right] T + v_i(n) \tag{3}$$

where $\alpha_i > 0$ is the sensitivity of the ith vehicle. τ_i is nonnegative integer, which represents the reaction time steps of the ith vehicle. It means that the vehicles' controllers take actions after $\tau_i T$ time. $V_i^{op}\left(y_i(n)\right)$ is the OV function which depends on a headway distance $y_i(n)$ between the $i-1$th and ith vehicle:

$$y_i(n) = x_{i-1}(n) - x_i(n) \tag{4}$$

The OV function has been given by [33]

$$V_i^{op}\left(y_i(n)\right) = \frac{vi^{\max}}{2}\left[1 + \overline{H}_{\text{sat}}\left(2\frac{y_i(n) - \eta_i}{\xi_i}\right)\right] \tag{5}$$