Physica A 511 (2018) 263–271

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Determination of preferential exponent α in random processes with a $1/f^{\alpha}$ power spectrum

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HIGHLIGHTS

- The principle of maximum information entropy may be used to analyzing the stability of random processes with $1/f^{\alpha}$ power spectra in model systems.
- The principle of maximum information entropy allows determining the values of the exponent α at which the $1/f^{\alpha}$ process is more stable.
- Spectral entropy has a minimum, which corresponds to a critical state of the system, at which statistical entropy has a maximum.

ARTICLE INFO

Article history:
Received 20 June 2018
Available online 26 July 2018

Keywords:
Nonlinear stochastic equations
$1/f$ noise
Nonequilibrium phase transitions
Entropy
Maximum entropy

ABSTRACT

The stability of fluctuation processes with $1/f^{\alpha}$ power spectra in systems of stochastic equations simulating coupled phase transitions has been analyzed with the use of the maximum information entropy principle. It is shown that in the class of processes described by a nonpotential system of model equations fluctuations with a $1/f$ power spectrum, i.e. with the exponent $\alpha = 1$, are preferential, Preferential for the class of potential systems under consideration are fluctuations with a $1/f^{\alpha}$ spectrum at $\alpha = 1.3$. The spectral entropy of the random processes has been calculated. Calculation of the spectral entropy makes it possible to investigate the stability of random processes directly from the power spectra, without calculating the probability density functions. The dependence of the spectral entropies on the amplitude of white noise has a minimum whose position corresponds to the critical behavior.

0. Introduction

Extremely large fluctuations may arise in various physical, chemical, biological and many other systems that are far from equilibrium (e.g. [1–7]). A considerable part of the energy of such fluctuations is connected with slow processes, and in a system one may observe large scale low-frequency pulsations, in which the frequency dependence of the power spectrum will look like $S(f) \sim 1/f^{\alpha}$, where the value of the exponent α lies in the range $0.8 \leq \alpha \leq 1.8$. The energy of fluctuations with a $1/f^{\alpha}$ spectrum accumulates at low frequencies, therefore the may be large-scale surges in a system. A characteristic feature of large fluctuations is a scale invariant probability density function, which has power "tails". The relaxation of such fluctuations also has a power form, as distinct from an exponential relaxation of fluctuations in equilibrium systems. Such a situation is observed at the thermodynamic critical point of a liquid – vapor phase transition. The scale invariance of fluctuations of thermodynamic quantities close to the critical point is determined by the conditions of convergence of properties of

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https://doi.org/10.1016/j.physa.2018.07.046
0378-4371/

different phases and requires a fine tuning and large relaxation times [8,9]. As distinct from the thermodynamic critical point, nonequilibrium processes with large fluctuations show their stable spatial and temporal scale invariance without a fine tuning of parameters. Therefore the origin of extreme large fluctuations is often connected with the concept of self-organized criticality [1,10,11], which describes the avalanche dynamics and is used for demonstrating a critical behavior in a large number of computer models. Large fluctuations are often described with the use of analogies turbulent flows [12], and fractional integration of white noise [13].

When the spectral density of fluctuations is strictly in inverse proportional to the frequency ($\alpha = 1$), a random process is often called $1/f$ noise, As an example of $1/f$ noise, one should first of all mention fluctuations that arise in a passage of an electric current through various media. Such fluctuations studied for 90 years and called flicker noise (e.g. [14–17]). In [18–21] large fluctuations with a $1/f$ power spectra are revealed in critical and transient processes of heat and mass transfer, such as boiling crisis, explosive boiling-up in liquid jets and ultrasonic cavitations.

There are many more examples of stationary random processes with $\alpha \neq 1$. The exponent α determined from experiments or mathematical models may take values that differ for the same class of processes. The question arises as to whether all random processes with a $1/f^{\alpha}$ power spectrum (with a different exponent α) are equally stable.

To the highest stability corresponds the maximum of the Gibbs– Shannon information entropy H determined by the probability density functions of stochastic variables P_n [22]:

$$H = -\sum_{n} P_n \log(P_n). \tag{1}$$

The index n refers to the sampling rate of the variables. The information entropy has properties of the Gibbs statistical entropy, but it also valid not only for physical systems, but also for a wider class of social, biological, communication and some other systems studied by statistical methods.

A characteristic feature of large fluctuations with a $1/f^{\alpha}$ power spectrum is a scale-invariant probability density functions, which have power "tails". In statistical mechanics the principle of the maximum Gibbs–Shannon entropy is used for the Gaussian distribution of fluctuations. It is assumed that in studying complex system characterized by a distributions with power "tails" the Gibbs–Shannon entropy is unusable because of the nonintegrability of formula (1) [23,24].

The solve the problem of calculation of the entropy of a distributions with power "tails", Tsallis [25] suggested deforming the logarithmic function in the expression for the entropy. The Tsallis entropy is determined by the expression

$$H^T = \frac{1}{1-q}\left(\sum_{n} P_n^q - 1\right). \tag{2}$$

The use of the entropy introduced by Renyi [26] has also been suggested for power probability density functions:

$$H^R = \frac{1}{1-q}\ln\sum_{n} P_n^q. \tag{3}$$

Both entropies H^T and H^R contain the dependence on the parameter q, whose value determines the concrete position of the maximum entropy and whose value is unknown.

In our opinion, a complex system cannot be characterized by a single distribution function of variables. A sufficiently detailed description of a complex system have to contain a system of nonlinear stochastic equations, which are in a master–slave hierarchy [27].

In the present paper the maximum information entropy principle is used to analyze the stability of fluctuation processes with a $1/f^{\alpha}$ power spectrum.

1. Maximum entropy of $1/f$ fluctuations at coupled phase transitions

Previously we showed the random processes with large fluctuations might arise in coupled nonequilibrium phase transitions under the action of white noise [20,27–29]. In this case use is equations. made of a system of nonlinear stochastic

$$\begin{aligned} \frac{d\varphi}{dt} &= -\varphi\psi^2 + \psi + \xi_1(t), \\ \frac{d\psi}{dt} &= -\varphi^2\psi + \lambda\varphi + \xi_2(t), \end{aligned} \tag{4}$$

where φ and ψ are dynamic variables and ξ_1 and ξ_2 are Gaussian –δ -correlated noises. We introduce the potential:

$$\Phi = \frac{1}{2}\varphi^2\psi^2 - \varphi\,\psi. \tag{5}$$

Under the transformation to the new dynamic variables: $\eta = (\psi+\phi)/2$ and $\theta = (\psi-\phi)/2$ the potential Φ will take the form:

$$\Phi = \frac{1}{2}\eta^4 - \eta^2 + \frac{1}{2}\theta^4 + \theta^2 - \eta^2\theta^2, \tag{6}$$