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A scalar measure tracing tree species composition in space or time

Bogdan M. Strimbu [a,*], Mihaela Paun [b,e], Cristian Montes [c], Sorin C. Popescu [d]

[a] College of Forestry, Oregon State University, Corvallis OR, USA
[b] Faculty of Administration and Business, University of Bucharest, Bucharest, Romania
[c] Warnell School of Forestry and Natural Resources, University of Georgia, Athens GA, USA
[d] Department of Ecosystem Science and Management, Texas A&M University, College Station, TX, USA
[e] National Institute of Research and Development for Biological Sciences, Splaiul Independentei, 296, Bucuresti 060031, Romania

HIGHLIGHTS

- We identify the conditions for a measure of composition for mixed species forest stands.
- We develop a scalar measure of species composition.
- The new measure operates in the L^1 space.
- We prove that the new measure is suitable for modeling growth and stand dynamics

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ABSTRACT

The tree species composition of a forest ecosystem is commonly represented with weights that measure the importance of one species with respect to the other species. Inclusion of weight in practical applications is difficult because of the inherent multidimensional perspective on composition. Scalar indices overcome the multidimensional challenges, and, consequently, are commonly present in complex ecosystem modeling. However, scalar indices face two major issues, namely non-uniqueness and non-measurability, which limit their ability to be generalized. The objective of this study is to identify the conditions for developing a univariate true measure of composition from weights. We argue that six conditions define a scalar measure of species mixture: (1) usefulness, (2) all species have equal importance, (3) all individuals have the same importance, (4) the measurements expressing importance of an individual are consistent and appropriate, (5) the function measuring composition is invertible, and (6) the function is a true-measure. We support our argument by formally proving all the conditions. To illustrate the applicability of the scalar measure we develop a rectilinear-based measure, and apply it in yield modeling and assessment of ecosystem dynamics.

1. Introduction

Forest ecosystems with multiple tree species are more resilient to environmental changes [1–3] and support a wider wildlife population than ecosystems with a single dominating tree species, but are not necessarily associated with superior economic outcomes [4]. The contradictory evaluation of the ecosystems services is not confined to multiple tree species

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[*] Corresponding author.

E-mail addresses: bogdan.strimbu@oregonstate.edu (B.M. Strimbu), mihaela.paun@faa.unibuc.ro (M. Paun), crmontes@uga.edu (C. Montes), s-popescu@tamu.edu (S.C. Popescu).

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ecosystems, but is amplified by the representation of species composition. To explicitly consider all tree species, the composition of a forest ecosystem is measured with weights, which assess the importance of each species [5,6] (some authors consider weights as a measure of abundance). Description of composition with weights is not new [7,8], but multidimensionality limits its abilities to formally compare different forest ecosystems or even the same ecosystem throughout time or space. Nevertheless, multidimensional representations of composition are attractive, as they describe a forest ecosystem from both the richness and the evenness perspectives [9]. Scalars, which are the unidimensional alternative to the multidimensional measures, are easy to interpret and implement [10]. Scalars are easy to use because of their similarity, which allows development of complex formulation without appealing to additional assumptions. (One can think that each scalar contributes to development of a multidimensional space.) Unfortunately, there are multiple scalar measures of composition, typically focused on heterogeneity [5,11,12], which exposes them to interpretations that are either subjective or driven by assumptions. Chief among the assumptions defining unidimensional measures is the equivalence to a true measure [13]. It is a simple exercise to prove that this assumption is not valid, as most scalar measures do not fulfill the sigma additivity condition [14]. Not being a true measure confines the univariate measures of composition to specific research areas, a particular case being ecosystem diversity [5].

The representation of species mixtures at different moments or locations using scalar metrics based on weights, such as the Simpson index or the Shannon index, can lead to identical values for different compositions (e.g., an ecosystem with two species that has at one moment the weight 70% and 30% and at another the same values but reversed has identical Shannon or Simpson index). Therefore, current scalar metrics of composition either introduce an additional factor, sometimes subjective, when comparing ecosystems with different importances for the same species, or relate ecosystems based on space rather than composition (i.e., the evolution is traced for one ecosystem only, regardless of the measure that quantifies species mixture). Consequently, a scalar measure of composition that allows generalization across ecosystems is needed (e.g., like canopy closure or stand density index).

Attempts to unify some of the measures of composition exist times in the literature; Hill [15], Pielou [16] or Patil and Taillie [17] being just some of the researchers that investigated compositional metrics. Patil and Taillie [17] answered formally some of the equivalence questions related to diversity indices, but they did not address the logical consistency of the measure, in the sense that the whole is not the sum of its parts. Subsequent studies by Heip et al. [18] and Jost [19] presented some fundamental properties of composition when expressed as diversity, but two questions remained unanswered: (1) is a measure of species composition a true measure, and (2) can different compositions be represented by distinct values (i.e., scalars)?

The necessity of a metric that surpasses the non-uniqueness property was advocated by Strimbu [20], who provided a solution and an example of a measure that supplies different values to distinct species associations. However, Strimbu [20] did not formalize a framework for measuring the composition of a forest ecosystems. Therefore, the main objective of the present study is to establish the necessary and sufficient conditions of a true measure for the composition of a forest ecosystems that would allow differentiation of distinct species associations. Additionally, we will develop a scalar measure of composition that can be used in growth and yield modeling and ecosystem dynamics. Identification of different species compositions uniquely with scalars will allow the inclusion of species associations besides the traditional attributes used in ecosystem modeling.

2. Methods

2.1. Theoretical background

Any measure of species composition simultaneously addresses two facets of a forest ecosystem. First, it identifies the scope, namely the species of interest. Second, it estimates the importance of each species [6,21]. From the latter perspective, species composition is a measure of biodiversity, as it incorporates both a measure of richness as well as a measure of abundance [5]. Many measures have been developed to describe the composition of an ecosystem [5], every measure justified by its utility. While usefulness triggers the creation of a measure, its construction should (1) fulfill the assumptions of a compositional measurement as presented by Peet [22], and (2) meet the defining conditions of a true measure, as formalized by Kolmogorov and Fomin [23]. Peet [22] listed three fundamental requirements for a measure of composition:

1. all species have equal importance (i.e., there is no attribute that ranks or distinguishes compositional relevant species, except the attribute on which the measure is based, such as canopy closure, total height, or diameter at breast height),
2. all individuals have equal importance (i.e., similar to the first assumption but applied to individuals, meaning there are no differences in the evaluation of individuals included in the measure), and
3. species abundance has been recorded using appropriate and compatible units.

The three requirements mentioned above are based on the hypothesis that the forest ecosystem under investigation is well defined and with no boundary effects. The first two requirements provide an operational and ethical perspective on the measure, while the third one points to some pitfalls that should be avoided when a composition measure is crafted. However, the foundations of a compositional measure stated by Peet [22] do not guarantee logical consistency (e.g., the whole is the sum of its parts). Consequently, additional requirements should be imposed to ensure the fulfillment of the laws of logic,