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Stochastic process with multiplicative structure for the dynamic behavior of the financial market

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HIGHLIGHTS

- The temporal series of the volatility and the return for the model is analyzed.
- The distribution of the returns is verified and the power of the long tail distribution gotten.
- The Hurst index is calculated using the rescaled range analysis.

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ABSTRACT

A stochastic model with multiplicative noise has been proposed as a mathematical model for the prices dynamics of the financial market. We have presented a model which allows us to test within the same framework the comparative explanatory power of rational agents versus irrational agents with respect to facts of the financial market. We calculate the long range memory of the model and studied the behavior of the long tail distribution of the cumulative distribution of probabilities for the model with additive and multiplicative noise.

1. Introduction

Microscopic dynamics containing multiplicative noise may be encountered in several dynamical systems. In general, the dynamical behavior is a common feature of many non-equilibrium systems which can be also characterized by power-law probability density functions [1]. The dynamics of prices of the market has been studied since some decades ago by Black and Scholes with their famous equation for the price dynamics of the European market [2]. Being in the last decades, there has been a large amount of data on this subject [3]. In general, the mathematical modeling of the stock market must simulate the market structure, trading mechanism and price dynamics. Mike and Farmer have made an empirical behavioral model (MF model) in an order-driven market to simulate the dynamics of stock price formation [4]. In following, Gu and Zhou [5] have modified the MF model by incorporating long memory into aggressiveness of incoming orders [6].

In a general way, socioeconomic systems are complex systems in which extreme events occur more frequently and exhibit complex behaviors [7,8]. They have been a great field for the application of concepts and mathematical methods for theoretical physics, used to treat complex systems [9–11]. An important model that has been used for the modeling of the financial market is the two-dimensional Ising model and its extensions [12–17]. An important quantity to study the dynamics of the financial market is the volatility. The study of the volatility in time series is not only crucial for revealing the underlined mechanism of the financial markets dynamics, but also useful for traders because it can help them to estimate

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the risk and optimize the portfolio [18]. It is well known that financial time series look unpredictable and their future values are essentially impossible to predict [3]. Another way for modeling the financial market is through stochastic differential equations [19–21]. Moreover, the modeling through a set of linear ordinary differential equations also has been employed in the literature for modeling the market [22].

An important thing in finances is the observation of scaling laws in financial markets that are the widespread power-law behavior exhibited by large price changes. This is corroborated for practically all types of financial data and markets [23–26]. The quantity of interest here is the relative price change or return. The statistical analysis of daily returns indicates a strong evidence for a hyperbolic behavior of the tails of the distribution, were is well known that the distribution of returns is given as $r(t) \equiv \ln S(t+\Delta t) - \ln S(t)$ [3] and the long tail cumulative probabilities distribution obeys to an inverse cubic-law $P(|r|) \sim |r|^{-\gamma}$, where $\gamma \sim 3$ is the tail exponent. The volatility is defined as the modulus of the return, $g(t) = |r(t)|$ [5,7,27–32]. In contrast, empirical analyses for other stock markets have unveiled power-law tail exponents other than the Lévy regime and the inverse cubic-law. Moreover, the fluctuating character of the volatility in financial markets has been considered as major responsible for change of the price dynamics [1,25].

In this work we intend to study the model of price dynamics given by Itô's stochastic differential equation with a multiplicative noise and a nonlinear polynomial interaction potential of the type ϕ^4. In general, the stock price is governed by stochastic differential equations with a linear interaction potential (i.e. without a term of type ϕ^4 in the potential) [19]. Here, we intend to study the more general case with a nonlinearity of the type ϕ^4 included. The case of this model with additive noise has been considered recently in [33]. Moreover, nonlinear terms in the stochastic equations have been proposed as a model for stock market fluctuations and crashes [34]. Where the linear case represents "risk neutral" case. Terms of type ϕ^4 included can represent the market when it is unstable, with an exponential rise or decay of the stock value, corresponding to a speculative bubble.

The plan of this paper is the following. In Section 2, we describe the stochastic model. In Section 3, we present the numerical results and in Section 4, we present our conclusions and final remarks.

2. The model

The model is described by the following Itô stochastic differential equation with a multiplicative white noise

$$dX(t) = \left[\alpha X(t) - \beta (X(t))^3\right] dt + \gamma (X(t))^{\delta} dW(t), \tag{1}$$

with $\delta \in [0, 1]$ and $dW(t)$ being the Winner increment. For $\beta = 0.0$, is well known that the model above gives a good fit to option prices crossing different strikes at a single expiration date [19]. In this case, $\gamma(X(t))^{\delta-1}$ can be interpreted as the volatility that is a decreasing function of the stock price. When one wishes to account for different volatilities implied by options expiring at different dates as well as different strikes, one needs to allow γ to depend on t as well as x. The function $\gamma(t, x)$ is called the volatility surface [19].

The associated Fokker–Planck equation to the above model is given as

$$\frac{\partial P(x,t)}{\partial t} = \frac{\partial[(\alpha x - \beta x^3)P(x,t)]}{\partial x} + \frac{\gamma}{2}\frac{\partial^2[x^{\delta}P(x,t)]}{\partial t^2}. \tag{2}$$

The potential of type ϕ^4 makes the probability density solution of the steady state in a non-Gaussian form, for $\beta \neq 0.0$. For $\beta = 0.0$, it assumes the Gaussian form. Moreover, Eq. (1) can be used to describe the behavior of a particle under action of an force given by $f(x) = -\beta x^3 + \alpha x$ plus a dissipative force $-b\dot{x}$, being b a damping constant plus a stochastic noise of the environment $\delta\xi(t)$, where we can take the limit of the inertial term negligible ($m \simeq 0$). Such model can represent a financial market upon different extreme conditions on the process, where the price tends to go for two minimums, represented by the double well. The case δ and $\varsigma = 0$ has been considered in the Ref. [34] and is nominated as Bouchaud–Cont–Langevin model for stock market fluctuations and crashes.

For $\beta = 0.0$, we can write the Eq. (1) as [21]

$$\frac{dX_t}{X_t} = (\alpha - \beta X_t^2)dt + \gamma X_t^{\delta-1} dW(t). \tag{3}$$

Using the Itô's differential given as

$$d(\ln X_t) = \frac{1}{X_t} dX_t + \frac{1}{2}\left(-\frac{1}{X_t^2}\right)(dX_t)^2 \tag{4}$$

and

$$\frac{dX_t}{X_t} = d(\ln X_t) + \frac{1}{2X_t^2}\gamma^2 X_t^{2\delta} dt, \tag{5}$$

we have

$$d(\ln X_t) = \left(\alpha - \beta X_t^2 - \frac{1}{2}\gamma^2 X_t^{2\delta-2}\right) dt + \gamma X_t^{\delta-1} dW_t.$$