



# Optimal control design for a class of quantum stochastic systems with financial applications

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## HIGHLIGHTS

- The state of a stochastic system is modeled as a quantum diffusion process via a new Quantum Stochastic dynamics.
- An existence and uniqueness solution theorem for QSDE is proved.
- The explicit optimal quantum control laws are designed.
- Spread evolution Modelling, more precisely and increasing portfolio managing are achieved.

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## ABSTRACT

The purpose of this paper is to design an optimal quantum controller for a class of stochastic systems with application in financial problems. Dynamics of the system is prescribed via a Quantum Stochastic Differential System (QSDES) with a quantum Brownian motion on a quantum probability space. A theorem for guaranteeing the existence and uniqueness of solutions to the QSDES is proved. Additionally, a new optimal stochastic control problem is formulated and based on the necessary optimality conditions, an optimal quantum control law is designed, explicitly. Four theorems and two lemmas, for facilitating the optimal controller design algorithm, are proved. Finally, for demonstrating the applicable results, two financial problems, Merton portfolio allocation and optimal pairs trading problem are simulated by using the presented method. As the simulation results indicate, portfolio optimal performances, minimum risk and maximum return, are achieved via presented method.

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## 1. Introduction

The general topic of optimal control theory is to find a control law for a given system such that the certain optimality criterions are achieved. In an optimal control problem, an optimal control signal controls a dynamical system for minimizing a given cost function [1]. The Bolza problem, Lagrange problem and the Mayer problem are the most famous optimal control problems [1]. There are many methods to solve these optimal control problems. These methods are classified into two categories, the direct and indirect approaches [2]. In the direct approaches, the dynamical system is discretized and then an optimal solution of the discrete problem is obtained. Among the direct approaches, finite element method for parameterizing problem [3] and finite difference method [4] for state equations of the problem are common methods. One of the advantages of these methods is that an explicit solution for the optimal control problems can be obtained. In the indirect

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approaches, the necessary conditions for optimality of the control problem are determined by using variational techniques, such as variational calculus [5] and Pontryagin's maximum principle [6]. Finally, the resulting equations are discretized and solved numerically [7]. There are two general indirect methods, the Dynamic Programming method, developed by Bellman, and the Pontryagin Maximum Principle method [8]. The Dynamics Programming Problems (DPP) are common problems in economics, finance and natural resource management. It is worth noticing that, the value functions and the optimal strategies (actions or controls) are solutions of a given optimal control problem that is solved by DPP approach. Also, solutions typically are calculated by numerical approximation techniques which are various in using complexity and computational requirements [9].

In practice, many control systems have uncertain or imperfectly known disturbances, that may be taken as random variables. In the study of deterministic control models, these disturbances are ignored [8]. In contrast, optimal stochastic control theory handles models in which the disturbances of the system are modeled as a random noise with a known probability distribution function [8]. The stochastic optimal control aim is to design the time path of random variables controlled by a desired control task such that a cost function of the system be minimum [10]. The stochastic control context may be discrete or continuous time frameworks. In the discrete time framework, the objective is that sum of expected values of a non-linear function be optimized, from the initial to the final time [2]. The non-linear objective function may be a quadratic function possibly. In continuous-time control, the state of the system is known for controller in any time. Also, the objective is that integral of a concave function of a state variable be maximum over a horizon from initial time to a terminal time  $T$  or maximum of this function at some future date  $T$  [11].

Depending on type of the cost function and system dynamics, there are various stochastic optimal control problems that can be solved by various methods. The first class of such problems is Linear Quadratic Regulator control problems. If the system dynamics is linear and the cost function is a quadratic then the solution is presented as a number of coupled ordinary differential equations (Ricatti equations) [12]. For instance, a chemical system that operates around a desired point in a state space can be maintained by LQR control. Also, this method is applied in engineering, widely. However, LQR control cannot model the complexity of intelligent behavior of agents or robots, perfectly [12].

In the framework of controlled diffusion, the dynamics of state of the system is modeled by a Stochastic Differential Equation (SDE) such that the resulting stochastic optimal control problems are formulated on finite or infinite horizon [13]. Note that, to describe different systems, there are different types SDEs such as the classical SDEs with respect to Brownian motion, Levy processes and Poisson point processes [14]. The stochastic optimal control problems can be solved by two approaches, using a partial differential equation Hamilton–Jacobi–Bellman (HJB) and using the Pontryagin Maximum Principle (PMP) which are based on the dynamic programming method and the calculus of variations that yields a pair of ordinary differential equations, respectively [13]. The classical dynamic programming method is adopted when the priori assumption of the smoothness of the value function is satisfied. This is not established in nature, necessarily. To avoid this drawback, a suitable formulation in viscosity solutions for dynamic programming equations was introduced [13,15]. Another approach, called the convex duality martingale method, is found by using of the Stochastic Maximum Principle (SMP), which drives a system with Forward or Backward Stochastic Differential Equations (FSDE/BSDE) [16]. These modern presentations of stochastic optimal control problems have been motivated by portfolio optimization problems solving [13,17].

In using the approaches which were mentioned above, all aspects (the dynamics, the environment and the cost) are considered as known aspects. On the other hand, it is trivial that the real and physical systems have several types of uncertainty, in nature. For instance, if the state variable  $X_t$  be unknown, then a probability distribution via  $p(X_t|Y_{0:t})$  is used instead of  $X_t$ , where  $Y_{0:t}$  denotes all previous observations in Bayesian method. Also, it is possible that the parameters of a stochastic differential equation (SDE) be unknown. In such cases, the learning techniques on finite/infinite horizon [11], the partial observability problems [18–20] or the joint inference and control problems [21] can be useful. Additionally, the expectation values of the utility function, in the dynamic programming approach, are computed. Thus, all states need to be observed and some tedious calculations must be done. The reinforcement learning approach encounters these intractability's [22,23].

So far, we conclude that a partial differential equation must be solved to obtain the solution of the stochastic optimal control problems. This is not an attractive option, in practice. In this case, although LQR approach is considered as a common approach (i.e approximate the problem by a linear quadratic problem which can be solved using the Ricatti equations) but the path integral methods can solve a class of non-linear and non-quadratic control problems, also [24]. Additionally, in path integral method, the non-linear HJB equation is transformed into a linear equation by a log transformation [18,25]. In consequence of the linear description, instead of the usual backward integration of the HJB equation one can compute the expectation values under a forward diffusion process. In fact, a stochastic integration on the trajectories is used to compute the expectation values. Additionally, the stochastic integration can be described by a path integral [26]. The path integral leads us to non-linear Kalman filters method [27] and it's contributions in hidden Markov models [28]. There are various methods to approximate this path integral such as Laplace approximation [24], Monte Carlo sampling [29] and variational approximation [30].

A quantum control system can be manipulated as a control system for achieving to a desired state in a Hilbert space [31,32], based on quantum mechanics as a flexible and powerful framework for solving the control problems. Also, in addition to the path integral approach, the quantum mechanics uses the operator transformations to describe the stochastic optimal control problems in a linear formalism. By applying the optimization method for classical control system to the derivation of Hamilton–Jacobi equation, the resulting HJ equation of stochastic system is equivalent to the Schrödinger

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