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Analytical solution of the space-time fractional hyperdiffusion equation

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Abstract

The so-called fractional hyperdiffusion equation is presented to develop a fractional derivative model of the transport of energetic particles. The fractional hyperdiffusion equation is defined in terms of Caputo and Riesz fractional derivatives for time and space, respectively. The solution is obtained by using the Laplace-Fourier transforms and given in terms of the M-Wright and Fox's H functions. Profiles of particle densities are illustrated for different values of space-fractional order.

Keywords: Fractional Calculus- Anomalous Diffusion- Energetic Particles

1. Introduction

In recent years the interest in developing fractional transport equations has increased considerably to study anomalous transport processes [1–3] occurring, e.g., in astrophysical plasmas as superdiffusion of energetic particles [4–6]. Superdiffusion is observed in many different physical systems and is characterized by the mean square displacement of the diffusing species $\langle z^2(t) \rangle \sim t^\gamma$ with $1 < \gamma < 2$ and is arising due to the existence of long range correlations in the dynamics generated by the presence of anomalously large particle displacements [7]. Rigorous derivations of fractional transport equations from mechanisms ex-

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