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Target control of edge dynamics in complex networks

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HIGHLIGHTS

- A framework is proposed to characterize the target control of edge dynamics.
- A method is proposed to approximate the minimum number of driver nodes.
- Target control efficiency is determined mainly by the degree distribution.
- Target edge set selected by the random scheme is easier to control.
- Real networks tend to choose high-degree and divergent nodes as driver nodes.

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ABSTRACT

Controlling edge dynamics in complex networks, which is relevant to many real systems, is significant in network science. However, the full control of the edge dynamics may be neither feasible nor necessary in some real systems with huge size and complexity. This practical situation motivates us to explore target control, that is, the full control of a preselected subset of edges. In this paper, an effective method is proposed to approximate the minimum number of driver nodes sufficient for target control of the edge dynamics in complex networks. The method allows us not only to analyze the target control efficiency, i.e., a unified metric of the propensity of an edge dynamics to be target controllable, but also to show the structural property of driver nodes. Evaluation of real networks indicates that the target control efficiency is determined mainly by the network's degree distribution. Simulation results and analytic calculations show that the edge dynamics in dense and homogeneous networks tend to have higher target control efficiency. Also, the target edge set selected by the random scheme is easier to control than that by the local scheme and the critical scheme. Furthermore, the analysis of the local structure of driver nodes shows that spare and inhomogeneous networks, which emerge in many real systems, tend to choose high-degree and divergent nodes as driver nodes.

1. Introduction

Complex networks, due to the interaction between their dynamic units, are relevant to a variety of natural, social and technological systems [1,2]. According to control theory [3], the dynamics in complex networks is controllable if, with a suitable choice of inputs, it can be driven from any initial state to any desired final state within finite time. How to control the dynamical processes occurring in complex networks is a challenge because classical control theory [4,5] is hard to be directly applied to complex networks. As a framework, the structural controllability was proposed to study

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the controllability properties of complex networks in [6–8]. Under this framework, Liu et al. [9] developed the structural controllability for the nodal dynamics, and offering a minimum inputs theorem to determine the minimum number of driver nodes. Subsequently, Nepusz et al. [10] introduced the edge dynamics, and found that the controllability properties of edge dynamics are significantly differ from that of nodal dynamics. The importance of the network controllability problem motivates researchers to explore various aspects of controllability properties of complex networks, and many advances in the field have been witnessed [11–22].

Most existing studies focus on researching the full control of a given network. However, many realistic network systems may be neither feasible nor necessary to be controlled fully. In view of this practical situation, Gao et al. [15] introduced the target control for the nodal dynamics and proposed a greedy algorithm to approximate the minimum number of driver nodes sufficient for target control. They found that heterogeneous networks have higher target control efficiency, and that the local target control, in general, is more efficient than the random target control. The target nodes are chosen randomly in the random target control, and be chosen from a local network neighborhood in the local target control. Iudice et al. [16] further studied the target control for the nodal dynamics, and provided an efficient numerical tool based on integer programming to calculate the minimum number of driver nodes sufficient for target control.

The previous studies of target control are performed based on the nodal dynamics. We still lack a framework to characterize the target control of the edge dynamics in complex networks. In this paper, an effective method is provided to approximate the minimum number of driver nodes sufficient for target control of edge dynamics. Specifically, the target control can be transformed into finding a subgraph that satisfies two conditions. Firstly, it contains all target edges. Secondly, it contains the minimum number of divergent nodes. Then an efficient method based on integer programming is proposed to find the subgraph. The method provides an effective control for the edge dynamics with any preselected target edge sets.

An important application of the method is to analyze the target control efficiency, i.e., a unified metric of the propensity of an edge dynamics to be target controllable. Simulation results and analytic calculations show that several target controllability properties of the edge dynamics remarkably differ from that of the nodal dynamics. Specifically, the edge dynamics in dense and homogeneous networks tend to have higher target control efficiency. While heterogeneous networks are target controllable with higher efficiency for the nodal dynamics [15]. For the edge dynamics, the target edge set selected by the random scheme is easier to control than that by the local scheme and the critical scheme. While the local target control is generally more efficient than the random target control for the nodal dynamics [15]. Furthermore, evaluation of real networks indicates that the target control efficiency is determined mainly by the network's degree distribution. As the degree distribution is easy to calculate for any networks, this result allows us to estimate efficiently the target control efficiency of a given network by its degree distribution. Another important application of the method is to explore the local structural property of driver nodes. The analysis of the driver nodes shows that spare and inhomogeneous networks, which emerge in many real systems, tend to choose high-degree and divergent nodes as driver nodes. This result will play a guiding role for the selection of driver nodes when lacking the ability to determine exactly the driver nodes in some real systems.

2. Target control

2.1. Model

The switchboard dynamics [10] was proposed to describe a dynamical process on the edges of a directed network $G(V, E)$. Let $\mathbf{x} = (x_1, x_2, \ldots, x_M)^T$ denotes the state vector comprised of the state of each edge. $\mathbf{y}_v^-$ and $\mathbf{y}_v^+$ denote the state vectors comprised of the states of the incoming and outgoing edges of node v, respectively. The state vector $\mathbf{y}_v^+$ can be influenced by the state vector $\mathbf{y}_v^-$, the vector of damping terms $\boldsymbol{\tau}_v$, and the input vector $\mathbf{u}_v$. So the equations governing the edge dynamics are described as follows,

$$\dot{\mathbf{y}}_v^+ = S_v \mathbf{y}_v^- - \boldsymbol{\tau}_v \otimes \mathbf{y}_v^+ + \sigma_v \mathbf{u}_v, \tag{1}$$

where $S_v \in \mathbb{R}^{k_v^+ \times k_v^-}$ is a so-called *switching matrix* with row number equaling the out-degree k_v^+ and column number equaling the in-degree k_v^- of node v. σ_v is 1 if node v is a driver node and 0 otherwise, and $\otimes$ denotes the entry-wise product of two vectors of the same size. A correspondence between the switchboard dynamics and the linear time-invariant dynamical system can be established by rewriting the switchboard dynamics in terms of x_i of the edges, yielding

$$\dot{\mathbf{x}} = (W - T)\mathbf{x} + H\mathbf{u}, \tag{2}$$

where $W \in \mathbb{R}^{M \times M}$ is the transpose of the adjacency matrix of the line graph $L(G)$ generated from the original graph G, in which w_{ij} is nonzero if and only if the head of edge j is the tail of edge i. $H \in \mathbb{R}^{M \times M}$ is a diagonal matrix where the ith diagonal element is σ_v if node v is the tail of edge i. $T \in \mathbb{R}^{M \times M}$ is a diagonal matrix composed of the damping terms of each edge. Note that the damping matrix T, which has no effect on the controllability [10], can be simply ignored.

Let $\mathcal{E} = \{e_1, e_2, \ldots, e_{|\mathcal{E}|}\}$ be the target edge set of size $|\mathcal{E}| = fM$, where $f = |\mathcal{E}|/M$ belongs to [0, 1]. The target control of the edge dynamics can be viewed as a special type of output control as follows,

$$\begin{cases} \dot{\mathbf{x}} = (W - T)\mathbf{x} + H\mathbf{u}, \\ \mathbf{y} = C\mathbf{x}, \end{cases} \tag{3}$$

where $C = [I(e_1), I(e_2), \ldots, I(e_{|\mathcal{E}|})]$ is the special output matrix. $I(i)$ denotes the ith row of an $M \times M$ identity matrix I.