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Uncertainty analysis of shear stress estimation in circular channels by Tsallis entropy

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HIGHLIGHTS

- A new approach is applied to estimate uncertainty of the shear stress estimation.
- Tsallis entropy has been employed for estimating the shear stress in open channels.
- The effect of normalization on the distribution of errors are examined.
- The proposed method can be used as an alternative in practical applications.

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ABSTRACT

Accurate prediction of the shear stress distribution is essential for the successful design of stable erodible-bed channels and for the sediment transport studies. Considerable attention in recent years has been given to the estimation of velocity distribution using entropy concept in open channels. Despite the importance of knowledge about shear stress distribution, there are very few studies on the application of the entropy methods for prediction of the shear stress distribution in open channels. The Tsallis entropy has been employed in this study for estimating the shear stress in open channels. In this approach, a pair of mean and maximum shear stresses are used to evaluate the shear stress distribution on the entire channel cross-section. We then calculated the prediction uncertainty of the shear stress obtained from the Tsallis entropy in a circular open channel. Moreover, the distribution of prediction error for the Tsallis approach is examined in two cases, both before and after data normalization. The quantitative results from this uncertainty analysis showed satisfactory results for the Tsallis entropy model for estimating shear stress in the entire section. The 95% Confidence Bounds (CB) are obtained for the shear stress distribution predicted by the model closely match the observed values.

1. Introduction

The importance of boundary shear stress has been shown in many practical hydraulic engineering applications. The knowledge of the shear stress (mean or local) in many hydraulic equations is required. However, channel bed shear stress is very difficult to measure directly under field condition [1]. Open channels with circular cross sections are one of the commonly used fragments in many water hydraulic conveyances. Knight and Sterling [2] carried out an extended

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experimental study on boundary shear stress distribution in circular channels and in circular open channels with flatbeds. Several studies referred to this experiments in order to validate their methods for circular channels or circular channels with flatbed [3–10].

Khodashenas et al. [6] performed a comparison between six approaches for estimation of boundary shear stress in open channel flow. Their results showed that each method could be applied to a particular case and certain hydraulic condition and there is no general approach which can estimate shear stress distribution in all cross section with high accuracy. Sterling et al. [7] developed a numerical model to solve Navier–Stokes equations based on large eddy simulation to simulate the instantaneous characteristics of boundary shear stress. Although their results can estimate the local shear stress, they concluded that the behavior of the shear stress is a complex phenomenon in open channels. Martinez-Vazquez and Sharifi [8] determined boundary shear stress distribution in open channels by using a pattern recognition technique. The authors stated that the robustness of the proposed method needs further studies specifically for the circular cross-section. Sheikh Khozani and Bonakdari [9] applies gene expression programming approaches to predict the distribution of boundary shear stress in circular open channels with and without the presence of sediment. The authors proposed an explicit equation, but a high number of parameters and complexity of equations limit its application. Sheikh Khozani et al. [10] studies the effect of different parameters on shear stress value by using extreme learning machines. Although, the results can predict the shear stress distribution with reasonable accuracy. They concluded further research might be carried out on modeling the shear stress distribution. All these studies show an accurate estimation of the shear stress distribution is a challenging task.

Chiu [11] employed Shannon entropy [12] concept based on probability in the modeling of hydraulic parameters of open channels. The author obtained commonly one-dimensional vertical shear stress distribution law for steady uniform flow in a wide channel by entropy. However, their study focused on velocity distribution. Following this study Chiu and several researcher performed extensive study on application of entropy concept for estimating velocity distribution [13–28]. Although, Chiu was the first researcher who applied the Shannon entropy for shear stress estimation. He did not perform further research on this subject in his next papers.

While researchers have more attention to the application of entropy to the estimation of velocity distribution in open channels and other parameters in water engineering [29–39], much less attention has been given to shear stress distribution evaluation. Sterling and Knight [40] carried out an attempt to predict the transverse boundary shear stress distribution in open channel using Shannon entropy. Although obtained results were acceptable accuracy compare with experiments, the authors declared the shear stress evaluation based on the entropy approach is generally disappointing. They proposed further development of the application of entropy concept for estimation of the shear stress distribution. Sheikh and Bonakdari [41] used power law in Shannon entropy for estimation of the shear stress distribution in circular and trapezoidal channels. This simplified equation successfully predicts the shear stress value for the studied cross-section and overcome the limitations of previous Shannon equation. Khozani and Bonakdari [42] estimated the shear stress values using Renyi entropy in circular channels. They referred to high precise of Renyi entropy in the prediction of shear stress values in circular and circular flatbed channels.

Tsallis [43] proposed a general form of the standard expression of the Shannon entropy which adequately describes the statistical features of complex systems in combination with different algorithms and package [44–50]. Tsallis entropy formulation contains an additional parameter that can be less susceptible to the probability distribution form [51,52]. Indeed, the Tsallis formalism naturally introduces a non-additive algebra parameter [53] that keeps the additivity of desirable variable (x) and generalizes other additive properties whose lack motivated the present paper to evaluate the Tsallis entropy uncertainty [52].

Luo and Singh [54] and Singh and Luo [46] applied Tsallis entropy successfully for deriving two-dimensional, and one-dimensional velocity distribution in open channel flows respectively. Cui and Singh [55,56] used some modifications to previous Tsallis entropy studies for estimation of the velocity distribution. Bonakdari et al. [57] developed a new formulation for evaluation of the shear stress distribution based on Tsallis entropy. Their results showed that the shear stress Tsallis based equation has more accuracy in comparison with Shannon entropy based equation. As the proposed Tsallis approach contains two Lagrange parameters that cannot be measured directly, involve some inherent degree of statistical uncertainty. Therefore, uncertainty analysis is required to prove the accuracy of the obtained shear stress values.

Misirli et al. [58] developed a Bayesian algorithm as sequential estimation methods for estimation of parameter and output uncertainty for watershed models. Bayesian estimation is known as a reliable method to quantify the inability of the model to produce correct predict that accurately reflect model parameter and predictive uncertainty [48,49,59,60]. Corato et al. [61] used the procedure proposed by Misirli et al. [58] in order to perform an uncertainty analysis of flow velocity estimated by Shannon entropy model. These authors showed that Bayesian estimation could be applied to quantify the inability of the model. Although several studies on hydrology and artificial intelligence techniques focused on determining the uncertainty of predicting models [62–66], no research has been carried out on deciding the uncertainty analysis of shear stress.

This study aimed to examine the uncertainty analysis of the shear stress obtained from the Tsallis entropy in the circular open channel and circular channel with a flatbed for different flow depths for the first time by improvement of uncertainty procedure proposed by Misirli et al. [58] and used with Corato et al. [61]. In the present paper, Non-Gaussian nature of the error distribution is normalized by Box–Cox transformation [67] that this method was not used in Misirli et al. and Corato et al. studies. Furthermore, in the present paper, four formal normality tests of Kolmogorov–Smirnov, Anderson–Darling, Shapiro–Wilk, and Lilliefors are utilized to evaluate data normalization in the before and after normalization. The errors of