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An extended lattice hydrodynamic model based on control theory considering the memory effect of flux difference

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HIGHLIGHTS

- A new model is proposed considering the memory effect of flux difference and delay feedback signal.
- The stability analysis is carried out based on control method.
- The mKdV equation is deduced to describe traffic congestion.

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ABSTRACT

Nowadays, the memory effect of drivers' behavior has been a hot topic in traffic flow research. In this paper, based on the lattice hydrodynamic model a new feedback control model is derived in a single-lane system. The memory effect of flux difference is considered in the new model to suppress the traffic jam. The critical condition of the model is analyzed by control method. The simulations are applied to verify the influence of feedback control signal on alleviating traffic jam. Besides, energy consumption simulation is designed in this paper. All the results demonstrate that the memory effect of flux difference model enhances the stability of traffic flow.

1. Introduction

In the past few decades, with the development of society, more and more people have possessed their own cars, and traffic problems have been a hot topic. Traffic congestion is the most attractive one which causes a great number of economic loss and gas emission. To reveal the formation of traffic jam and reduce the traffic congestion, a variety of traffic flow models [1–29], such as the lattice hydrodynamic model [10–16], the car-following model [17–23], are studied by experts.

Car-following model describes the relationship of longitudinal vehicles, which was firstly introduced by Pipes [19] et al. In 1961, a famous car-following model was proposed by Newell [20] with the consideration of optimal velocity (OV) function, and a graphic description about the OV function was given. In 1995, Bando [30,31] et al. carried out a classic car-following model called optimal velocity model (OVM) to solve the problem of infinite acceleration in Newell's model. The OVM reveals that the driver adjusts the velocity based on the headway distance. In 2006, Zhao et al. proposed coupled-map (CM) car-following model [32,33], and the velocity feedback control [34,35] was introduced for the first time. According to Zhao's research, it is helpful to suppress the traffic congestion by considering the feedback control signal in models.

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The lattice hydrodynamic model was firstly constructed by Nagatani [36] who incorporated the ideas of car-following theory and discretized the hydrodynamic model built by Kerner and Konhauser [37]. The lattice hydrodynamic model is intuitive to depict traffic jamming transition. Owing to the superiority of the lattice hydrodynamic model, more and more researchers have been deeply attracted by lattice hydrodynamic model to analyze the macroscopic traffic flow. Because the lattice hydrodynamic model is coupled with the density wave method, there were few scholars focusing on control method. Until 2015, Ge [38] proposed a new lattice hydrodynamic model considering the feedback signal called flux difference to suppress the traffic jam in view of modern control theory [39]. In the same year, Redhu [40] presented delayed-feedback control (DFC) model with the consideration of driving behavior. Subsequently, effects of density change rate difference were considered as a feedback signal by Li [41]. Recently, Wang [42] focused on the influence of optimal flux for forward looking sites in his new model.

During the actual driving condition, drivers always adjust their driving behavior by their driving experience. In 1959, Herman [43] found that drivers remain the memory of historic information during the driving process. In his paper, the memory effect has a significant influence on drivers, so many scholars [44–46] pay attention to it. From above the papers, it can be revealed that memory effect can improve the traffic flow stability. But most of researchers focused on the memory at a time point, as we know, drivers decide their behavior during a period of time. In 2017, Cheng [47] proposed the view of memory effect during a period of time. Therefore, a new lattice hydrodynamic model is introduced considering the memory effect of flux difference during a period of time by control method, and the energy consumption [48] of new model is investigated to verify the stability of system.

The outline of the paper is listed as follows. In Section 2, the basic lattice hydrodynamic model is derived, and we introduce the control signal for lattice hydrodynamic model. In Section 3, a control signal will be added into the basic lattice hydrodynamic model and feedback control theory is used to analyze the stability conditions. In Section 4, several numerical simulations including energy consumption analysis are carried out to verify the theoretical results. Conclusions are given in Section 5.

2. Basic model

The basic lattice hydrodynamic model was carried out associated with the advantages of hydrodynamic models and car-following model to describe the evolution of traffic flow on a single-lane highway. The equations are written as follows [37]:

$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0 \\ \partial_t(\rho v) = a\rho_0 V(\rho(x+\delta)) - a\rho v, \end{cases} \tag{1}$$

where ρ is current density at time t and ρ_0 is average density. δ is the average space headway and x is the position of local lattice, so $\rho(x+\delta)$ is the current density on the position of $x+\delta$. Especially ρ_0 and δ can be expressed: $\rho_0 = \frac{1}{\delta}$. a represents the sensitivity coefficient of driver. Nagatani [31] creatively put forward the discretization method to explain the equation as follows:

$$\begin{cases} \partial_t \rho_j + \rho_0 \left(\rho_j v_j - \rho_{j-1} v_{j-1}\right) = 0 \\ \partial_t(\rho_j v_j) = a\rho_0 V(\rho_{j+1}) - a\rho_j v_j, \end{cases} \tag{2}$$

where j represents the location of lattice. In Eq. (2) ρ_j and v_j are current density and current velocity. Simplify the equation and consider the feedback signal:

$$\begin{cases} \partial_t \rho_{j+1} + \rho_0 \left(q_{j+1} - q_j\right) = 0 \\ \partial_t(q_j) = a\rho_0 V(\rho_{j+1}) - aq_j + u_j, \end{cases} \tag{3}$$

where control signal u_j is listed as follows, and k is the control parameter, τ_0 represents memory time:

$$u_j = k \int_{t-\tau_0}^{t} [q_j(s) - q_{j+1}(s)]ds. \tag{4}$$

Nagatani [25] also proposed the new optimal velocity function of lattice hydrodynamic model:

$$V(\rho) = (v_{max}/2)\left[\tanh(1/\rho - 1/\rho_c) + \tanh(1/\rho_c)\right], \tag{5}$$

where v_{max} represents the maximum velocity of vehicle on this road and ρ_c is safety density.

3. Control scheme

In this section, we take the memory effect of flux difference into account, and research the influence of new feedback signal on traffic flow by control method. We assume that first lattice's flux and density are not affected by others in a steady state. The steady equation is described as follows:

$$[\rho_n(t), q_n(t)]^T = [\rho_n^*, q_n^*]^T. \tag{6}$$