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Complexity–entropy causality plane based on power spectral entropy for complex time series

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HIGHLIGHTS

- We propose complexity–entropy causality plane based on power spectral entropy.
- The power spectral entropy is derived from Fourier transformation and is free of parameters.
- Time series generated from different classes of systems can be clearly distinguished in our plane.
- The plane can determine the start–stop time and classification of fault signals corresponding to bearing vibration signals.

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ABSTRACT

The complexity–entropy causality plane based on permutation entropy is a powerful tool to discriminate signals from different systems. In this paper, we combine traditional statistical complexity measure and power spectral entropy and construct complexity–entropy causality plane in frequency domain. The power spectral entropy is derived from Fourier transformation, so some features that are obscure in time domain can be extracted in frequency domain. Comparing to permutation entropy, this method is free of parameters. Several time series generated from different classes of systems are analyzed to demonstrate the measure. Results show that these signals can be clearly distinguished in our plane. Then by adding sinusoidal abnormal signal into original one, the abnormal information can be efficiently detected. Finally, we apply it to bearing vibration signals. Empirical consequences illustrate that the start–stop time and classification of fault signal can be clearly determined.

1. Introduction

Investigation on complex system is a hot topic in recent years. Researchers have introduced lots of complexity measures, such as Kolmogorov complexity [1], fractal dimensions [2], Lyapunov exponents [3] and entropies [4,5]. Moreover, inspired by the definition of entropy, many entropy methods are proposed to analyze time series [6,7]. However, some of the methods rely on specific algorithm or tuning parameters, which may lead to false identification. Bandt and Pompe had posed this issue and proposed a new method attempting to work out the former problems, named permutation entropy [8]. This method has been extensively utilized in nonlinear dynamic systems [9–11].

Nevertheless, deriving from the definition of entropy, the completely ordered process probability distribution is concentrated in a certain state, and only a small amount of information can be obtained to describe its system behavior, so

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the information is considered to be the smallest. On the other hand, the maximum random process is fully disordered, and the probability of any state is equal, so the quantity of information reaches the largest. Completely ordered and maximum random systems, as two simple systems, are at two extremes in the "information" measure (maximum and minimum). So there are some limitations only considering the "information" to describe the complexity of systems. The distance between the uniform distribution (equal probability distribution) and system probability distribution is a reasonable measure of complexity, and disequilibrium is just the characteristic of the probability distribution, giving a view of probabilistic hierarchy of system. The value of disequilibrium would be different from zero if the privileged, or more probable status exists. The combination of "information H" and "disequilibrium Q" is taken as a statistical complexity measure C known as López–Mancini–Calbert (LMC) complexity measure, that is $C = HQ$, where H is in accordance with thermodynamic entropy and Q is the quadratic distance [12]. Corresponding to the different values of the entropy measure S and the disequilibrium Q, different statistical complexity measures are generated, such as Shiner–Davidson–Landsberg (SDL) statistical complexity [13]. There are a large number of possible physical structures between two specific cases, completely ordered and maximum random process. And the degree of statistical complexity can be reflected by the potential system probability distribution, which derives a series of statistical complexity measures and are used to reveal the complex dynamics that are implicit in the system.

By integrating permutation entropy and complexity measure, Rosso et al. proposed complexity–entropy causality plane (CECP) which is a powerful tool that describes the dynamic characteristics of the system [14–17], involving LMC complexity measure [12], SDL complexity measure [13] and Jensen–Shannon divergence. This method can better characterize the complexity of the physical structure of the system. By applying Jensen–Shannon divergence instead of Euclidean distance, this generated statistical complexity measure has the intensive characteristic detected in lots of thermodynamic quantities, which can also better reflect the key details of the dynamic characteristics of the system, and distinguish between different degrees of periodicity and chaos, whereas this kind of information cannot be recognized by the randomness measure. And its applications greatly spread over various scientific community for distinguishing noise, chaotic system and stochastic process [18–24].

Here we combine traditional complexity–entropy causality plane (CECP) with power spectral entropy, aiming to analyze signals in frequency domain. The proposed algorithm is novel and rather simple. What should be done is just changing permutation entropy into power spectral entropy. Unlike permutation entropy, power spectral entropy (PSE) is free of any parameters. PSE interprets the spectrum structure of signal in frequency domain, in other words, time uncertainty. If the distribution of energy is more uniform in the whole frequency domain, the signal has more complexity and thus the value of PSE is higher; whereas the narrower the spectrum peak is, the smaller the value of PSE is, which means that the system is more concussive and less complex. And the statistical complexity measure C is also redefined, which estimates the distance between power spectrum distribution of signal and uniform power spectrum distribution.

The rest of the paper is organized as follows. Section 2 introduces the methodologies of statistical complexity measure and power spectral entropy. Then we integrate these measures and present the complexity–entropy causality plane based on power spectral entropy and sliding window. In Section 3, we select several different kinds of processes to validates the effectiveness of proposed method, and then analyze the power spectrum of original signal and signal adding sinusoidal information. Section 4 displays the empirical application in rolling bearing data. Finally, the conclusions are drawn in Section 5.

2. Methodology

2.1. Statistical complexity measure

Statistical complexity can be defined to describe a system with a simple structure but complex dynamical characteristics and can also reveal complex patterns that are implicit in its inner dynamics [25]. At the same time, the statistical complexity considers that there are two opposing extremities in the nonlinear dynamic system, that is, completely order and maximum randomness. In both cases, the system structure is very simple, only zero statistical complexity. Between these two specific cases, there are a large number of possible physical structures, which can be reflected by the potential system probability distribution characteristics [26].

For a given nonlinear system with any arbitrary discrete probability distribution $P = \{p_i, i = 1, 2, \ldots, N\}$ with N possible statuses, the well-known Shannon information theory is described as [4]:

$$S[P] = -\sum_{i=1}^{N} p_i log(p_i) \tag{2.1}$$

The value of $S[P]$ quantifies the complexity of the system with some degree. If $S[P] = 0$ we can predict certainly that all the possible outcomes i whose probability is set by p_i will actually happen. On the other hand, the uncertainty reaches maximum when the distribution is uniform, that is, $S[P_e] = S_{max} = logN$ where $P_e = \{1/N, 1/N, \ldots, 1/N\}$.

Another complexity measure is "disequilibrium", denoted by Q, which depicts the distance between a specified probability distribution and the equilibrium probability distribution. And the disequilibrium Q is defined in terms of the extensive