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Long-time behavior of a stochastic logistic equation with distributed delay and nonlinear perturbation

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HIGHLIGHTS

- A stochastic logistic model with distributed delay and nonlinear perturbation is studied.
- We transfer the stochastic model with strong kernel or weak kernel into an equivalent system.
- We establish sufficient and necessary criteria for extinction of the population.
- We use Markov semigroup theory to obtain the existence of a unique stable stationary distribution.

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ABSTRACT

In this paper, we study a stochastic logistic model with distributed delay and nonlinear perturbation. We first use the linear chain technique to transfer the one-dimensional stochastic model with strong kernel or weak kernel into an equivalent system with degenerate diffusion. Then we establish sufficient and necessary criteria for extinction of the population with probability one. Moreover, since the diffusion matrix is degenerate, the uniform ellipticity condition is not satisfied. The Markov semigroup theory is used to obtain the existence and uniqueness of a stable stationary distribution. We show that the densities of the distributions of the solutions can converge in L^1 to an invariant density. The results show that the larger white noise can lead to the extinction of the population while the smaller white noise can guarantee the existence of a stable stationary distribution which leads to the stochastic persistence of the population. Numerical simulations are provided to demonstrate the theoretical results.

1. Introduction

Recently, delay differential equation has received much attention due to its important applications in biology and medicine. And infinite delay has been widely introduced into equations used in mathematical biology since the works of Volterra [1] to translate the cumulative effect of the past history of a system. Many researchers (see e.g. [2–5]) have studied the dynamical properties of logistic models with distributed delay. In [2], Rasmussen et al. considered the illustrative

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example of generalized logistic equations where the carrying-capacity effect is modeled by a distributed-delay effect. They showed that if the delay is sufficiently large a supercritical Hopf bifurcation occurs. This mirrors the situation when there is just a point delay. Li [3] presented a theorem for existence and uniqueness of almost periodic solutions for logistic equations with infinite delay. Yang et al. [4] investigated a non-autonomous logistic type impulsive equation with infinite delay. They obtained sufficient conditions which guaranteed the permanence of solutions. Bernard et al. [5] considered the single independent variable case where the delay is linearly distributed. They derived stability conditions for some standard distribution forms. Especially, Seifert [6] proposed a single species non-autonomous logistic system with infinite delay and the corresponding autonomous form can be expressed as follows

$$\frac{dx(t)}{\mathrm{d}t} = x(t)\left(a - b\int_{-\infty}^{t} K(t-\tau)x(\tau)d\tau\right),\ t \geq 0, \tag{1.1}$$

where $x(t)$ is the population density of the species at time t, the parameter a denotes the intrinsic growth rate and $\frac{b}{a}$ denotes the portion of delayed dependence of the growth rate. The parameter $\frac{a}{b}$ is the environmental carrying capacity of system (1.1). Seifert studied system (1.1) together with the following initial condition $x(t) = \phi(t)$, $t \leq 0$; $\phi(t)$ is continuous and bounded on $(-\infty, 0]$ to $[0, \infty)$ with $\phi(0) > 0$. Here the weight function $K : [0, \infty) \to [0, \infty)$ is piecewise continuous and satisfies

$$\int_0^{\infty} K(s)ds = 1,\ \sigma \equiv \int_0^{\infty} sK(s)ds < \infty.$$

For the distributed delay, MacDonald [7] initially revealed that it is reasonable to use Gamma distribution

$$K(t) = \frac{t^n c^{n+1} e^{-ct}}{n!}$$

as a kernel, where $c > 0$ and n is a nonnegative integer. The average time lag is

$$T = \int_0^{\infty} sK(s)ds = \frac{n+1}{c}.$$

In particular, the strong kernel

$$K(s) = c^2 s e^{-cs},\ c > 0$$

and the weak kernel

$$K(s) = ce^{-cs},\ c > 0$$

are often used (see Cushing [8]). The average time lags for the strong and weak kernels are

$$T = \frac{2}{c}$$

and

$$T = \frac{1}{c},$$

respectively. These two kinds of kernels have been widely used in biological systems, such as population systems (see e.g. [9–11]) and epidemiology [12] and neutral networks (see e.g. [13,14]).

On the other hand, in the real world, population systems are inevitably influenced by the environmental noise (see e.g. [15–19]). May [20] revealed the fact that due to the environmental noise, the birth rate, carrying capacity, competition coefficient and other parameters involved in the system should exhibit random fluctuation to a greater or lesser extent. Hence several authors introduced white noise into deterministic systems to reveal the effect of environmental noise on the population dynamics (see e.g. [21–31]). In particular, Mao [22] has obtained a surprising result: In some cases, the white noise not only can destabilize a system but also can stabilize a system. These interesting results reveal the important effect of the white noise on population systems.

As an extension of system (1.1), in this paper, we adopt a different approach to introduce random perturbation to the intrinsic growth rate a, that is,

$$a \to a + \alpha(1 + x(t))\dot{B}(t),$$

and so $adt \to adt + \alpha(1 + x(t))dB(t)$, where $\dot{B}(t)$ is the white noise, namely, $B = (B(t), t \geq 0)$ is a real-valued standard Brownian motion, $\alpha^2 > 0$ denotes the intensity of the white noise. Motivated by works referred above, the stochastic logistic model with nonlinear perturbation can be expressed as follows

$$dx(t) = x(t)\left(a - b\int_{-\infty}^{t} K(t-\tau)x(\tau)d\tau\right)\mathrm{d}t + \alpha x(t)(1 + x(t))dB(t). \tag{1.2}$$