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Pareto's 80/20 rule and the Gaussian distribution

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HIGHLIGHTS

- 80/20 rule fits a Gaussian distribution with a standard deviation twice the mean.
- This finding represents large characteristic variations in our society and nature.
- The rule can be also referred to as the 25/5, 45/10, 60/15, or 90/25 rule, etc.
- Our result suggests the existence of implicit negative contributors.

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ABSTRACT

The statistical state for the empirical Pareto's 80/20 rule has been found to correspond to a normal or Gaussian distribution with a standard deviation that is twice the mean. This finding represents large characteristic variations in our society and nature. In this distribution, the rule can be also referred to as, for example, the 25/5, 45/10, 60/15, or 90/25 rule. In addition, our result suggests the existence of implicit negative contributors.

1. Introduction

Pareto's 80/20 rule states that roughly 80% of all effects stem from 20% of all causes for many events, which conceptually contrasts the contribution of the vital few with that of the trivial many [1–3]. This rule has been applied in a variety of fields, such as economics [4–7], biology [8,9], ethology [10,11], and civil engineering [12,13], where its validity and usefulness have been demonstrated. Mathematically, the 80/20 rule is often interpreted as an instance of the Pareto distribution [14–16]. However, the power law of the Pareto distribution is originally a model intended to represent the probability of a variable exceeding a certain threshold value, and is also known as the survival or tail function. Meanwhile, many distributions pertaining to events in our society and nature, to which the 80/20 rule is often applied, more commonly follow the normal or Gaussian distribution. In other words, it may be more intuitive to assume a distribution with a peak around the average or mean in order to discuss the 80/20 rule, rather than the monotonic Pareto distribution. Therefore, in this short note we present an analysis of Pareto's 80/20 rule based on the Gaussian distribution.

2. Theory and calculation methods

The normal or Gaussian probability distribution $f(x)$ based on the central limit theorem is described as

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}, \quad (1)$$

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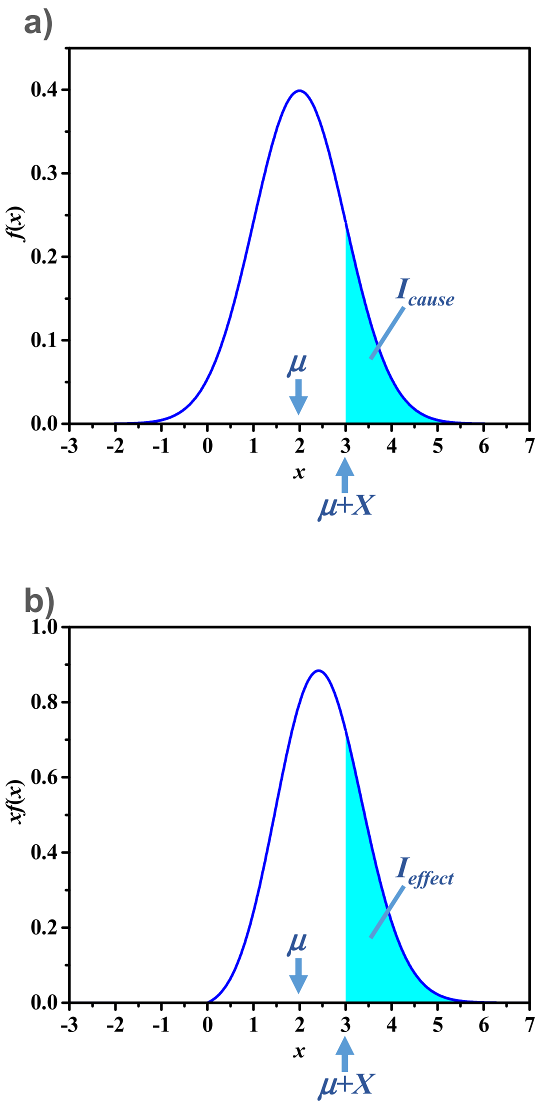

Fig. 1. Example of Gaussian distribution (a) $f(x)$ and (b) $xf(x)$ along with its corresponding I_{cause} and I_{effect} regions for $\sigma = 1$, $\mu = 2$.

where σ and μ are the standard deviation and the mean, respectively. To analyze and discuss Pareto's 80/20 rule, we define the cause and effect integrated fractions, respectively I_{cause} and I_{effect}, as

$$I_{cause} \equiv \frac{\int_{\mu+X}^{\infty} f(x)\,dx}{\int_{-\infty}^{\infty} f(x)\,dx} = \int_{\mu+X}^{\infty} f(x)\,dx, \tag{2}$$

$$I_{effect} \equiv \frac{\int_{\mu+X}^{\infty} xf(x)\,dx}{\int_{-\infty}^{\infty} xf(x)\,dx} = \frac{1}{\mu}\int_{\mu+X}^{\infty} xf(x)\,dx. \tag{3}$$

In popular folklore, the 80/20 rule includes such claims as that 20% of a population own 80% of the wealth, that 20% of the books in a library account for 80% of the circulation, that 20% of a business customers bring in 80% of its revenue, that 20% of all software features account for 80% of all software use, and so on. The quantity I_{cause} is a proportion of the population; the quantity I_{effect} is the proportion of the total income that they get. Or I_{cause} is a proportion of the population of books in a library and I_{effect} is the corresponding proportion of all circulation. And so on. Here, we note that

$$\int_{-\infty}^{\infty} f(x)\,dx = 1, \tag{4}$$