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Nonlinear analysis for a modified continuum model considering driver's memory and backward looking effect

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HIGHLIGHTS

- A new continuum model is developed considering driver's memory and backward looking.
- Applying the linear stability theory, the new model's linear stability is obtained.
- Through nonlinear analysis, the KdV–Burgers equation is derived.

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ABSTRACT

Account for the "integral form of optimal velocity changes with memory" and "backward looking" effect, an extended car-following model is proposed in this paper. Through the relation between macro and micro variables, the extended car-following is transformed to a new continuum model. According to the linear stability theory, the linear stability's condition of the presented model is obtained. The modified KdV–Burgers equation is established through non-linear analysis to describe the spreading behavior of the traffic density wave near the neutral stability line. The numerical simulation results show that the "backward looking" effect can improve the traffic flow's stability and the "integral form of optimal velocity changes with memory" can relieve the traffic congestion.

1. Introduction

In recent years, the problem of traffic congestion has become more and more popular in the society. In order to solve these serious traffic problems, many scholars have come up with many different models to study it. Many researches have proposed many traffic models [1–11] to describe the multitudinous traffic flows congestion phenomena, for example, the car-following model [12–23], the hydrodynamic model [6,24–32], the continuum model [33–39] and so on.

In 1955, Lighthill and Whitham [8,9] made a great contribution to the continuum models for the first time, and later Richards [10] drew the parallel conclusion independently (in short, the LWR model). After that, the LWR model was improved by Payne et al. [11]. In the LWR model, the relationship among the three basic parameters of the fluid is built by:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0, \tag{1}$$

where ρ, t, v and x respectively represent the density, time, speed and space. The model shows that the number of vehicles on the road section was conserved, but it cannot be applied to complex traffic flow such as the small disturbance.

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In 1971, Payne [11] established a new high-order continuum traffic flow model, which can be described the actual traffic effectively. What is more, this model can be used to analyze the traffic phenomena, such as the phantom traffic jam and the stop-and-go traffic. The dynamic equation is:

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}=-\frac{\alpha}{\rho T}\frac{\partial \rho}{\partial x}+\frac{v_e-v}{T}, \tag{2}$$

where T is relaxation time, and α is the anticipation coefficient.

In order to explain the traffic phenomenon exactly, Bando et al. [7] proposed the optimal velocity model (OVM)

$$\frac{dv_n(t)}{dt}=a\left[V\left(\Delta x_n(t)\right)-v_n(t)\right], \tag{3}$$

which solves the problem of boundless acceleration and can simulate a lot of qualitative characteristics of some actual traffic flow, such as the stop-and-go phenomenon, the congestion phenomenon and so on. In the actual traffic environment, for the sake of keeping the safety of the driver, the driver will observe the information of the car through the rear view of mirror. At the same time, the driver will have a memory of the traffic situation. In the last few years, a variety of neoteric factors are considered such as the driver's forecast effect [34], the "backward looking" effect [40–45], optimal velocity changes with memory [46], signal effect for energy dissipation [47] and so on.

As we all know, during the driving process, when the driver observes the deceleration, even the headspace is relatively large, but he will keep speed steady in advance. On the contrary, the leading car accelerates in a situation that the headspace is small, and the driver will increase the desired speed.

In view of this, Nakayama et al. [40] established a novel model which called the backward looking optimal velocity model (BLOV model). Then, Ge et al. [42] added the "backward looking" effect to the lattice hydrodynamic model in 2008. Sun et al. [43] considered the "backward looking" effect in the car-following model in 2011. In 2013, Yang et al. [44] derived the "backward looking" effect with a new multiple car-following model. In 2015, Yu et al. [45] considered "velocity changes with memory feedback" effect in the dynamics of connected cruise control systems. Peng et al. [48] devoted nonlinear analysis to a new car-following model accounting for the optimal velocity changes with memory. Cheng et al. [49] considered "multiple optimal velocity" effect to the extended macro traffic flow model in 2017. Cheng et al. [50] considered the "driver's memory" effect in the continuum model in 2017. Jin et al. [51] devoted nonlinear density wave investigation for an extended car-following model considering driver's memory and jerk. Qin et al. [52] added the "flux change rate and delay feedback signal" effect to the lattice hydrodynamic model. In 2018, Zhu et al. [53] devoted a new car-following model for autonomous vehicles flow with mean expected velocity field. In 2018, Tang et al. [54] proposed a cellular automation model accounting for bicycle's group behavior. In 2018, Tang et al. [55] modeled the electric bicycle's lane-changing and retrograde behaviors. Ou et al. [56] established an extended two-lane car-following model in 2018. In 2018, Yu et al. [21] established a relative velocity difference model. Xin et al. [57] considered "variable vehicular gap policies" effect to the extended car following model in 2018.

But the "optimal velocity changes with memory" and "backward looking" effect in the continuum model was seldom researched previously. As a result, an extended continuum model incorporating the "integral form of optimal velocity changes with memory" and "backward looking" effect is proposed in this paper.

The paper is organized as follows: In Section 2, the theory of linear stability is given and then the stability analysis is obtained. In Section 3, nonlinear stability analysis is carried out, which includes the KdV–Burgers equation is derived and its corresponding solution is obtained. In Section 4, the numerical simulation is conducted to validate the nonlinear result. In the end, the conclusion is given in Section 5.

2. The continuum model and linear stability analysis

Some scholars [37–39] find optimal speed model cannot solve the problem that the vehicle congestion when the vehicle speed is too large and the distance between the two vehicle heads is smaller than the safe distance. In 2001, Jiang et al. proposed the FVDM on the basis of previous study to solve the problem. The model equation [12] is:

$$\frac{dv_n(t)}{dt}=a\left[V\left(\Delta x_n(t)\right)-v_n(t)\right]+\lambda\Delta v_n, \tag{4}$$

where $\Delta x_n=x_{n+1}-x_n$, which denotes the distance difference between the $n+1$th and the nth vehicles; $\Delta v_n=v_{n+1}-v_n$, which denotes the speed difference between the two cars; a denotes driver's distance sensitivity coefficient. In fact, it is impossible for the vehicle to regress in the process of advancing, so this paper takes $a>0$; λ is the sensitivity coefficient of driver to speed difference; $V\left(\Delta x_n(t)\right)$ is optimal velocity function.

In 2012, Sun [21] incorporated the driver's backward looking into the new model. The model control equation is:

$$\frac{dv_n(t)}{dt}=a\left[pV_F\left(\Delta x_n(t)\right)+(1-p)V_B\left(\Delta x_{n-1}(t)\right)-v_n(t)\right]+\lambda\Delta v_n. \tag{5}$$

Based on the above analysis, we proposed an extended model which considered integral form of optimal velocity changes with memory and backward looking as the major factor in traffic flow.