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Lee–Yang zeros and two-time spin correlation function

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HIGHLIGHTS

- We study two-time correlation function for probe spin interacting with spin system.
- We find that zeros of the correlation correspond to the Lee–Yang zeros.
- New possibility to observe the Lee–Yang zeros experimentally is shown.

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ABSTRACT

The two-time correlation function for probe spin interacting with spin system (bath) is studied. We show that zeros of this function correspond to zeros of partition function of spin system in complex magnetic field. The obtained relation gives new possibility to observe the Lee–Yang zeros experimentally. Namely, we show that measuring of the time dependence of correlation function allows direct experimental observation of the Lee–Yang zeros.

1. Introduction

After works of Lee, Yang [1,2] and Fisher [3] analysis of partition function zeros are considered as a tool of studying properties of phase transitions in different systems [4].

Partition function can posses zeros if a parameter in a hamiltonian is generalized to the complex plane. These zeros are known as Lee–Yang zeros. Lee and Yang studied zeros of partition function for ferromagnetic Ising model with complex magnetic field [2]. The scientists proved the theorem that all zeros are purely imaginary. This theorem holds for any Ising-like model with ferromagnetic interaction [5] (see also [6–8]). It was also shown that distribution of zeros determines equation of state, analysis of behavior of partition function zeros near positive real axis are useful in studies of phase transitions [1]. In 1965 Fisher generalized the Lee–Yang result to the case of complex temperature [3]. Zeros of partition function in this case are known as Fisher zeros.

Studies of zeros of partition function have attracted much attention. It is worth noting that these studies are important fundamentally. Zeros of partition function fully determine the analytic properties of free energy and are very useful for investigating thermodynamical properties of many-body systems, studying phase transitions. Many works have been devoted to analysis of partition function zeros for spin systems (see, for instance, [9–16] and references therein). Also zeros of partition function of Bose systems (see, for instance, [17–20]) and Fermi systems (see, for instance, [21,22]) were examined. The Lee–Yang theory was used in studies of equilibrium and as well nonequilibrium processes. Applicability of Lee–Yang

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zeros analysis for a nonequilibrium phase transitions was investigated in the works [23–25]. Also, analysis of Lee–Yang zeros were used in studies of trajectory phase transitions [26–28] phase transitions in a molecular zipper [29]. To examine dynamical phase transitions the Fisher zeros were considered [30–32].

Because of difficulties in the experimental realization of many-body systems with complex parameters, for a long time studies of Lee–Yang zeros were restricted by theoretical considerations. In 1998 an experimental access to examine the density function of zeros on the Lee–Yang circle for a ferromagnet was provided [9]. Later in paper [10] (see also [11]), the authors showed the possibility of direct experimental observation of Lee–Yang zeros for partition function of spin system on the basis of analysis of decoherence of probe spin. The authors of [13] reported direct experimental observation of Lee–Yang zeros. The observation was done by measuring quantum coherence of a probe spin with is coupled to spin bath of Ising-type. Report on experimental determination of the dynamical Lee–Yang zeros was presented in [33]. The observation of the Lee–Yang zeros of a nonequilibrium process was done on the basis of measurements of the dynamical activity of trajectory.

In our recent paper [20] we showed the possibility of experimental observation of Lee–Yang zeros for interacting Bose gas, considering time-dependent correlation function. In [34] we found that zeros of time-dependent correlation functions of q-deformed Bose gas are related with the Fisher zeros.

In the present paper we relate zeros of two-time correlation function of probe spin with zeros of partition function of spin system (bath) with which probe spin is interacted. Comparing to the results of our previous papers [20,34] where we found expressions which relate zeros of time-dependent correlation functions of q-deformed Bose gas with Fisher zeros [34], zeros of two-time correlation functions of interacting Bose gas with Lee–Yang zeros [20], in the present paper we show that zeros of two-time spin correlation function are related directly with the Lee–Yang zeros. This direct relation in principle gives a new possibility for experimental observation of Lee–Yang zeros for spin systems in addition to that presented in [10,13]. We would like to note that in papers [10,13] the Lee–Yang zeros were studied on the basis of analysis of decoherence of probe spin. As far as we know the possibility of direct experimental observation of Lee–Yang zeros of spin system on the basis of analysis of zeros of the two-time correlation function of probe spin is firstly proposed in the present paper.

The paper is organized as follows. In Section 2 we give the preliminary information about the system under consideration. In Section 3 we find relation of zeros of two-time spin-1/2 correlation functions with Lee–Yang zeros. The possibility of simple experimental realization of considered system is shown in Section 4. Conclusions are presented in Section 5.

2. Hamiltonian of system under consideration

We are interested in study of the Lee–Yang zeros of spin-1/2 system under magnetic field with sufficiently general Hamiltonian

$$H = H' - h\sum_{i=1}^{N}\sigma_j^z, \tag{1}$$

We consider only one restriction on the H', namely that H' commutes with the total spin

$$[H', \sum_{i=1}^{N}\sigma_j^z] = 0. \tag{2}$$

There is not restriction on H' to be composed of only σ_z terms. Hamiltonian H' can be Ising Hamiltonian $H' = -\sum_{jj'} J_{jj'}\sigma_j^z\sigma_{j'}^z$, Heisenberg Hamiltonian $H' = -\sum_{jj'} J_{jj'}(\boldsymbol{\sigma}_j\boldsymbol{\sigma}_{j'})$ or some other that satisfy (2). Pauli operators σ_j^α ($\alpha = x, y, z$, or 1, 2, 3) are related with the spin operators $s_j^\alpha = \hbar\sigma_j^\alpha/2$ and satisfy the following commutation relations

$$[\sigma_j^\alpha, \sigma_{j'}^\beta] = 2i\delta_{jj'}\epsilon^{\alpha\beta\gamma}\sigma_j^\gamma. \tag{3}$$

In addition these operators satisfy anticommutation relation

$$\{\sigma_i^\alpha, \sigma_i^\beta\} = 2\delta^{\alpha\beta}. \tag{4}$$

Here $\epsilon^{\alpha\beta\gamma}$ is antisymmetric tensor and $\delta_{jj'}$, $\delta^{\alpha\beta}$ are Kronecker symbols.

We include into consideration probe spin-1/2 coupled to the considered system (bath) which is described by Hamiltonian (1), with the probe–bath interaction

$$H_{int} = -\lambda\sigma_0^z\sum_{j=1}^{N}\sigma_j^z, \tag{5}$$

where σ_0^z corresponds to the probe spin, λ is a coupling constant. So, the total Hamiltonian is

$$H_T = H + H_0 + H_{int} = H' - h\sum_{i=1}^{N}\sigma_j^z - h_0\sigma_0^z - \lambda\sigma_0^z\sum_{j=1}^{N}\sigma_j^z, \tag{6}$$