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Galvano- and thermo-magnetic effects at low and high temperatures within non-Markovian quantum Langevin approach

I.B. Abdurakhmanov [a], G.G. Adamian [b,*], N.V. Antonenko [b,e], Z. Kanokov [b,c,d]

[a] *Curtin Institute for Computation, Department of Physics, Astronomy and Medical Radiation Sciences, Curtin University, Perth, WA 6845, Australia*
[b] *Joint Institute for Nuclear Research, 141980 Dubna, Russia*
[c] *National University, 700174 Tashkent, Uzbekistan*
[d] *Institute of Nuclear Physics, 702132 Tashkent, Uzbekistan*
[e] *Tomsk Polytechnic University, 634050 Tomsk, Russia*

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ABSTRACT

The quantum Langevin formalism is used to study the charge carrier transport in a two-dimensional sample. The center of mass of charge carriers is visualized as a quantum particle, while an environment acts as a heat bath coupled to it through the particle–phonon interaction. The dynamics of the charge carriers is limited by the average collision time which takes effectively into account the two-body effects. The functional dependencies of particle–phonon interaction and average collision time on the temperature and magnetic field are phenomenologically treated. The galvano-magnetic and thermo-magnetic effects in the quantum system appear as the results of the transitional processes at low temperatures.

1. Introduction

The behavior of solid matter under the influence of external fields at low temperature is one of the interesting topics in solid-state physics [1–3]. The external field may be an electric field, magnetic field, optical signal, or temperature gradient. These fields modify some electronic properties, such as the carrier concentration and the carrier mobility. Besides the carrier mobility, the electric current is also affected by the magnetic field which deflects its direction and leads to a nonzero cross voltage (the classical Hall effect) linearly proportional to the field strength [1–4]. The oscillator nature of the longitudinal magneto-resistance of bismuth sample at low temperature, known as the Shubnikov–de Haas effect, has been also observed in the presence of very intense magnetic fields [1,3,5,6]. The effect is more pronounced at low temperatures where the amplitude of oscillations is significantly larger. The first experimental study of the influence of electric fields of the order of 100 mV cm^{-1} on the Shubnikov–de Haas magneto-resistance oscillations in n-InSb sample has been reported in Ref. [5]. In addition to the shift of the extremes to higher magnetic fields, a decrease of oscillation amplitudes with electric field has been observed. The Integer Quantum Hall Effect (IQHE) in the GaAs-$Al_{0.3}Ga_{0.7}As$ heterostructure has been discovered [7,8] at strong external electromagnetic fields and very low temperature. The quantization of conductivity surprisingly occurred in a certain two-dimensional electron gas (2DEG) under the influence of a strong magnetic field. In the IQHE, the Hall conductance

* Corresponding author.
E-mail address: adamian@theor.jinr.ru (G.G. Adamian).

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σ_{xy} has a stepwise dependence (the appearance of plateau) on the strong external magnetic field. At these plateau, σ_{xy} is quantized as $\sigma_{xy} = ie^2/(2\pi\hbar)$, $i = 1, 2, \ldots$, while the longitudinal conductivity σ_{xx} nearly vanishes. The vanishing σ_{xx} implies the absence of dissipation. This is another hallmark of the IQHE. Also, the Fractional Quantum Hall Effect (FQHE), where the Hall conductivity is quantized in fractional multiples of $e^2/(2\pi\hbar)$, has been discovered [9,10].

The theoretical models in Refs. [11–15] for describing the IQHE and FQHE have been developed. The combination of a random potential created by impurities in a sample and strong magnetic field gives rise to the special coexistence of localized and extended electron states. As known, the Fermi level lies in the energy gap (mobility gap) free from the extended states and the change of the electron density or the magnetic field can only result in different occupations of localized states which do not affect the conductivity. Based on these findings the appearance of the conductivity quantization has been explained. The general approach, which explains the quantization as well as the integer quantized values, has been developed later with the scaling theory. This approach also describes correctly the regions where the conductivity is not quantized [11]. For the theoretical explanation of the FQHE, the wave functions have been introduced [12] to describe the incompressible quantum states and explain the small but experimentally prominent class of fractions 1/odd. It turned out that the quasiparticle excitations are the charge/fluxcomposites with a fractional charge and statistics, also known as dubbed anions. The special properties of charge/fluxcomposites have been used in Refs. [13–15] to construct two so-called hierarchies, sets of Hall fractions for which the incompressible ground states could be found. These hierarchies are able to reproduce all fractions observed, but also yield many fractions that have never been measured. The striking universality in the manifestation of the quantum Hall effect attracted large attention, not only in solid state physics but also in high energy physics. The extensive mathematical methods of topological field theory [16] and infinite dimensional algebras [17–19] have been applied to the IQHE and FQHE. As found in several independent works, the description of incompressible quantum states exploits the theory of chiral edge currents [17]. The Quantum Hall ground states and quasiparticle excitations have been described in terms of representations of the infinite-dimensional algebras [17–19].

The aim of the present work is to treat the classic and quantum Hall effect as well as the Shubnikov–de Haas effect within the same model. The basic idea of our model is the following. In the electric current, we determine the time-dependent number of electrons with given momentum at a certain location. We consider the center of mass of charge carriers with a positive charge $e = |e|$ as a quantum particle coupled to the environment (heat bath) through the particle–phonon interactions. Solving the second order Heisenberg equations for the heat bath degrees of freedom, the generalized non-Markovian Langevin equations are explicitly obtained for a quantum particle. The memory effects in these equations results from the coupling to the environment. The dynamics of the charge carriers is restricted by the average collision time. The functional form of the particle–phonon coupling strength and the average collision time on the temperature and magnetic field are phenomenologically treated.

The paper is organized as follows. In Section 2, we introduce the Hamiltonian of the system and solve the generalized non-Markovian Langevin equations for a quantum particle. The electric and thermal conductivities are derived in two-dimensional systems. Note that the quantum Langevin approach or the density matrix formalism has been widely applied to find the effects of fluctuations and dissipation in macroscopic systems [20–47]. The main assumptions of the model are discussed. The model developed is used in Section 3 to describe the experimental data on the classic and quantum Hall and Shubnikov–de Haas effects. A summary is given in Section 4.

2. Non-Markovian Langevin equations with external magnetic and electric fields

2.1. Derivation of quantum Langevin equations

Let us consider two-dimensional motion of a quantum charged particle in the presence of heat bath and external constant electric $\mathbf{E} = (E_x, 0, 0)$ and magnetic fields $\mathbf{B} = (0, 0, B)$. The total Hamiltonian of this system is

$$H = H_c + H_b + H_{cb}. \tag{1}$$

The Hamiltonian H_c describes the collective subsystem (quantum particle) with effective mass tensor and charge $e = |e|$ in electric and magnetic fields:

$$H_c = \frac{1}{2m_x}[p_x - eA_x(x, y)]^2 + \frac{1}{2m_y}[p_y - eA_y(x, y)]^2 + eE_x x = \frac{\pi_x^2}{2m_x} + \frac{\pi_y^2}{2m_y} + eE_x x. \tag{2}$$

Here, m_x and m_y are the components of the effective mass tensor, $\mathbf{R} = (x, y, 0)$ and $\mathbf{p} = (p_x, p_y, 0)$ are the coordinate and canonically conjugated momentum, respectively, $\mathbf{A} = (-\frac{1}{2}yB, \frac{1}{2}xB, 0)$ is the vector potential of the magnetic field, and the electric field E_x acts in x direction. For simplicity, in Eq. (2) we introduce the notations

$$\pi_x = p_x + \frac{1}{2}m_x\omega_{cx}y, \qquad \pi_y = p_y - \frac{1}{2}m_y\omega_{cy}x$$

with frequencies $\omega_{cx} = \frac{eB}{m_x}$ and $\omega_{cy} = \frac{eB}{m_y}$. The cyclotron frequency is $\omega_c = \sqrt{\omega_{cx}\omega_{cy}} = \frac{eB}{\sqrt{m_x m_y}}$.