Contents lists available at ScienceDirect

# Physica A

journal homepage: www.elsevier.com/locate/physa

# A delayed-feedback control method for the lattice hydrodynamic model caused by the historic density difference effect

Guanghan Peng<sup>a,b</sup>, Shuhong Yang<sup>c,\*</sup>, Hongzhuan Zhao<sup>d</sup>

<sup>a</sup> College of Physics and Electronics, Hunan University of Arts and Science, Changde 415000, China

<sup>b</sup> College of Physical Science and Technology, Guangxi Normal University, Guilin 541004, China

<sup>c</sup> School of Computer Science and Communication Engineering, Guangxi University of Science and Technology, Liuzhou, 545006, China

<sup>d</sup> College of Architecture and Transportation Engineering, Guilin University Of Electronic Technology, Guilin, 541004, China

## HIGHLIGHTS

- A delayed-feedback control pattern is proposed by considering historic density difference effect.
- Control theory is applied to get the linear stability condition involving historic density difference effect.
- Numerical simulations verify that the historic density difference effect improves traffic stability.

## ARTICLE INFO

Article history: Received 19 March 2018 Received in revised form 18 May 2018 Available online xxxx

Keywords: Traffic flow Lattice hydrodynamic model Delayed-feedback control method Traffic congestion

# ABSTRACT

A delayed-feedback control pattern is brought forward for lattice hydrodynamic model of traffic flow with the consideration of the historic density difference effect. We utilize the modern control theory to obtain the linear stability conditions with the consideration of feedback control signal, which shows that the stability of traffic flow is closely related to the information of the historic density difference. Furthermore, numerical simulations affirm that the delayed-feedback control signal can suppress the traffic congestion efficiently. © 2018 Elsevier B.V. All rights reserved.

#### 1. Introduction

Traffic problem has more and more attracted considerable scholars' attention since it is the key factor that affects the circulation of commodities. To solve traffic jams, a variety of traffic models have been sprung up such as the car-following models, the cellular automation models, the lattice hydrodynamic models, and coupled map models etc. [1–19]. Recently, to suppress the traffic jams, control theory is introduced to construct traffic flow model. On the basis of the coupled-map (CM) car-following model [20,21], there occur a series of improved CM models to restrain the traffic congestion with the consideration of different control signals [22–28]. He et al. [29] further proposed feedback control scheme based on CM model for two-lane traffic flow. Very recently, the control theory was taken into lattice hydrodynamic model [31,32]. However, until now, the information of the historic density difference has not been investigated in lattice hydrodynamic model. In real traffic, the information of historic density plays an important role on the traffic flow. Therefore, a new delayed

\* Corresponding author. E-mail addresses: pengguanghan@163.com (G.H. Peng), netysh\_gxut@126.com (S.H. Yang).

https://doi.org/10.1016/j.physa.2018.06.097 0378-4371/© 2018 Elsevier B.V. All rights reserved.







feedback control signal in this paper is employed for lattice hydrodynamic model to increase the stability of the traffic flow by taking into account the historic density difference effect.

The outline of this paper is organized as follows: in Section 2, we bring forth a new delayed feedback control signal resulted from the information of historic density difference for lattice hydrodynamic model. In Section 3, by applying control theory, we will obtain the linear stability condition. Numerical simulations are carried out for lattice hydrodynamic model with and without control signal in Section 4. Finally, the conclusions are given in Section 5.

### 2. The extended model

In 1998, Nagatani proposed an original lattice hydrodynamic model as follows [18,19]:

$$\partial_t \rho_j + \rho_0(q_j - q_{j-1}) = 0 \tag{1}$$

$$\partial_t(q_j) = a[\rho_0 V(\rho_{j+1}) - q_j] \tag{2}$$

where  $\rho_0$ ,  $\rho_j$  and  $q_j$  respectively show the average density, the local density and local flux.  $a = 1/\tau$  is the sensitivity of a driver.  $V(\rho)$  means the optimal velocity function described by [30]

$$V(\rho) = (v_{\text{max}}/2)[\tanh(1/\rho - 1/\rho_c) + \tanh(1/\rho_c)]$$
(3)

where  $\rho_c$  is the safety density. Therefore, a number of extended lattice models [33–38] were come up accounting for different traffic factors based on the Nagatani's idea. However, the historical density difference effect has not been investigated in previous lattice models. Therefore, we proposed a new delayed-feedback control decided by the historical density difference effect to alleviate the traffic congestion on roads as follows:

$$\partial_t \rho_j + \rho_0(q_j - q_{j-1}) = 0 \tag{4}$$

$$\partial_t(q_j) = a[\rho_0 V(\rho_{j+1}) - q_j] + k \frac{\rho_{j+1}(t-1) - \rho_{j+1}}{\rho_0}$$
(5)

where *k* is the delayed-feedback gain. It is obvious that a delayed-feedback control signal is designed into the flow evolution equation by considering the difference between delayed density state and current one of site j+1. When k = 0, the model is same as Nagatani's model [18,19]. In order to be convenient for following analysis, we rewrite the Eqs. (4) and (5) as follows:

$$\partial_t \rho_{j+1} + \rho_0(q_{j+1} - q_j) = 0 \tag{6}$$

$$\partial_t(q_j) = a[\rho_0 V(\rho_{j+1}) - q_j] + k \frac{\rho_{j+1}(t-1) - \rho_{j+1}}{\rho_0}$$
(7)

#### 3. Linear stability analysis

In this section, linear stability analysis is executed to investigate the historical density difference effect by means of control theory.  $[\rho_j, q_j]^T = [\rho^*, q^*]$  is supposed as the steady-state uniform flow solution with the desired density and flux of the traffic flow. Hence, the new controlled system can be linearized nearby the steady state as follows:

$$\partial_t \rho_{j+1}^0 + \rho_0 (q_{j+1}^0 - q_j^0) = 0 \tag{8}$$

$$\partial_t(q_j^0) = a[\rho_0 \Lambda \rho_{j+1}^0 - q_j^0] + k[\rho_{j+1}^0(t-1) - \rho_{j+1}^0(t)]/\rho_0$$
(9)

where  $\rho_{j+1}^0 = \rho_{j+1} - \rho^*$ ,  $q_j^0 = q_j - q^*$ ,  $q_{j+1}^0 = q_{j+1} - q^*$ ,  $\Lambda = \left. \frac{\partial V(\rho_{j+1})}{\partial \rho_{j+1}} \right|_{\rho_j = \rho_0}$ . Then, we take Laplace transform for Eqs. (8) and (9) to win

$$sP_{j+1}(s) - \rho_{j+1}(0) + \rho_0[Q_{j+1}(s) - Q_j(s)] = 0$$
(10)

$$sQ_j(s) - q_j(0) = a[\rho_0 \Lambda P_{j+1}(s) - Q_j(s)] + k[P_{j+1}(s)e^{-s} - P_{j+1}(s)]/\rho_0$$
(11)

where  $P_{j+1}(s) = L(\rho_{j+1})$ ,  $Q_j(s) = L(q_j)$ ,  $Q_{j+1}(s) = L(q_{j+1})$ , L indicates the Laplace transform. By combining Eqs. (10) and (11), one deduces the flux equation as below:

$$Q_{j}(s) = \frac{-a\rho_{0}^{2}\Lambda - k(e^{-s} - 1)}{d(s)}Q_{j+1}(s) + \frac{a\rho_{0}\Lambda + k(e^{-s} - 1)/\rho_{0}}{d(s)}\rho_{j+1}(0) + \frac{s}{d(s)}q_{j}(0)$$
(12)

where d(s) represents the characteristic polynomial:  $d(s) = s^2 + as - a\rho_0^2 \Lambda - k(e^{-s} - 1)$ . Then, one obtains the transfer function G(s) as below:

$$G(s) = \frac{-a\rho_0^2 \Lambda - k(e^{-s} - 1)}{d(s)}$$
(13)

Download English Version:

# https://daneshyari.com/en/article/7374799

Download Persian Version:

https://daneshyari.com/article/7374799

Daneshyari.com