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Short note on conditional collapse of self-gravitating system with positive total energy

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HIGHLIGHTS

- We show a finite time gravitational collapse of Smoluchowski–Poisson system.
- We consider a highly clustered system inducing a large gravitational contribution to total energy.
- Highly clustered self-gravitating particles provide a sufficient blow-up condition for positive total energy.

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ABSTRACT

In this study, we first show a finite time collapse of a three-dimensional Smoluchowski–Poisson system in the micro-canonical ensemble (i.e., μSP system) with positive total energy. Using a precise estimation of the bound of the gravitational contribution to total energy, we provide blow-up conditions on the initial state of the μSP system. The conditions indicate that highly clustered self-gravitating particles in the μSP system induce a finite time collapse, even for positive total energy.

1. Introduction

Thermodynamics of a self-gravitating system involves various phenomena, such as gravitational collapses, phase transitions, etc. [1–6]. Smoluchowski–Poisson (SP) systems confined to an isolated medium have been studied [7–10] in order to theoretically analyze the thermodynamics of self-gravitating particles. Indeed, SP systems have displayed many thermodynamical features as at most metastable equilibrium states, basin of attraction and collapse (i.e., finite time blow-up) [11,12]. An SP system in the canonical ensemble (i.e., constant temperature) collapses even in two dimensions below a critical temperature [13]. In the micro-canonical ensemble (i.e., conserved energy), time-varying temperature, $\theta(t)$, suppresses the collapse of the SP system in two dimensions [9], whereas the system collapses under an energy below a critical energy in three dimensions [9,10]. According to [11,12], the collapse of a three-dimensional SP system in the micro-canonical ensemble (hereafter, μSP system) can be suppressed by imposing a sufficiently large energy. In particular, it was shown in [11,12] that radially symmetric solutions to a μSP system exist globally in time if energy is sufficiently large (or equivalently if there is sufficiently high initial temperature) for a given initial density. However, whether or not a μSP system can collapse for a relatively small energy that is nonetheless greater than a critical energy is still unresolved. Since there are no global entropy maxima [14,15], an asymptotic behavior of μSP system strongly depends on initial states [10,16]. Depending on whether an initial state lies in a basin of attraction, a μSP system can relax towards a metastable equilibrium state (local

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entropy maximum) or collapse (gravothermal catastrophe) [1,6,15]. Therefore, if the energy is larger than a critical energy, a collapse of μSP system occurs only if a gravitational attraction overcomes a stabilizing effect.

We focus on a conditional finite time collapse of a μSP system with positive total energy. To the best of our knowledge, blow-up results of the μSP system in a star-shaped domain have been provided under non-positive total energy [9]. Particularly, we revisit the μSP system having highly clustered self-gravitating particles with positive total energy. We prove finite time blow-up of solutions to the μSP system by a precise estimation of the bound of the gravitational contribution to total energy.

We consider a μSP system confined to the domain, $\Omega \subset \mathbb{R}^3$:

$$\begin{cases} \partial_t n = \nabla \cdot \left(\nabla n + \dfrac{n}{\theta(t)} \nabla \varphi \right) & \text{in } \Omega \times (0, T), \\ \Delta \varphi = n \end{cases} \tag{1}$$

which is subjected to non-negative initial density and temperature conditions, $n(x, 0) = n_0(x) \geq 0$ and $\theta(0) = \theta_0 > 0$, with $\int_\Omega n_0(x)dx = 1$. Here, $n(x, t)$, $\varphi(x, t)$, and $\theta(t)$ denote the density of self-gravitating particles, Newtonian potential generated by the particles, and time-varying temperature, respectively. No flux boundary condition is assumed, so that the total mass, $\int_\Omega n(x, t)dx$, is conserved:

$$\frac{\partial n}{\partial \nu} + \frac{n}{\theta(t)} \frac{\partial \varphi}{\partial \nu} = 0 \qquad \text{on } \partial\Omega \times (0, T). \tag{2}$$

The potential, φ, satisfies *free* condition [9];

$$\varphi(x, t) = (\Gamma * n)(x, t) = \int_\Omega \Gamma(x - y) n(y, t) dy, \tag{3}$$

with the Newtonian kernel, $\Gamma(x) = -1/(4\pi |x|)$. According to the energy conservation law, the total energy, $E \in \mathbb{R}$, is constant, and consists of thermal (kinetic) and potential (gravitational) contributions:

$$E = \kappa\theta(t) - I(t) \tag{4}$$

with the specific heat of the particles, $\kappa > 0$, and

$$I(t) := -\int_\Omega n(x, t)\varphi(x, t)dx = \int_{\Omega\times\Omega} \frac{n(x, t)n(y, t)}{4\pi |x - y|} dxdy, \quad t \in [0, T).$$

Thus, E is not time-varying and is fixed at $t = 0$ by (4), with the initial temperature, θ_0, and density, $n_0(x)$. For the sake of simplicity, we assume that the domain boundary, $\partial\Omega$, is smooth and star-shaped with respect to $\mathbf{0}$, holding

$$\inf_{x\in\partial\Omega} x \cdot \nu \geq 0 \tag{5}$$

for the unit outer normal vector, $\nu = \nu(x)$.

For the existence of local-in-time weak solutions to (1)–(3) satisfying $n \geq 0$, we refer to [9,11,17]. We note that a possible singularity of term, $1/\theta(t)$, in (1)–(2) can be removed by the universal lower bound of temperature [9,11,12]:

$$\theta(t) \geq \lambda\theta_0 > 0 \quad \text{for} \ \ t > 0. \tag{6}$$

More precisely, combining the Boltzmann entropy relation of (1)–(3),

$$\frac{d}{dt}\left[\int_\Omega n \log n \, dx - \frac{\kappa}{2} \log\theta\right] = -\int_\Omega n \left|\nabla\left(\log n + \frac{\varphi}{\theta}\right)\right|^2 dx \leq 0, \tag{7}$$

with Jensen's inequality $\left[\left(\int_\Omega n \, dx\right) \log\left(\int_\Omega n \, dx\right) \leq \int_\Omega n \log n \, dx\right]$ yields (6) with

$$\lambda = \exp\left(-\frac{2}{\kappa}\int_\Omega n_0 \log n_0 \, dx\right). \tag{8}$$

Before stating our main theorem, let us recall the known result on blow-up to clarify our contribution. Biler and Nadzieja [9] proved finite time blow-up of weak solutions for (1)–(5) by adopting a moments argument that has been broadly used to show blow-up of SP type systems (see [7,18,19] and references therein). Indeed, they [9] found that the second-order momentum of the density,

$$w(t) := \int_\Omega |x|^2 n(x, t) \, dx \geq 0, \tag{9}$$

evolves in time, satisfying the following equality:

$$\frac{d}{dt} w(t) = -(\kappa - 6) + \frac{E}{\theta(t)} - 2\int_{\partial\Omega} n(x, t) \, x \cdot \nu \, d\sigma. \tag{10}$$