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Nonlinear behavior of plasma: Connection with nonextensive statistics

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HIGHLIGHTS

- Nonextensive hydrodynamic equations and Zakharov equations are derived by moment equation and two time-scale methods, respectively.
- The conservation quantities and nonlinear entity collapse scalar law are obtained, from which we find that the conservation energy is relevant to the nonextensive parameter but momentum as well as angular momentum and the number of plasmon are not affected by the nonextensivity of system.
- The self-similar collapse solution of nonextensive Zakharov equations is also presented.
- we demonstrate that the nonlinear entity collapse scalar law is relevant to the nonextensive parameter and especially it allows the existence of three dimensional stable and one dimensional collapse nonlinear entity, which is significantly different from the case of Maxwellian distribution.
- In the extensive limit, all the results obtained in the framework of Maxwellian are reproduced

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ABSTRACT

Nonextensive hydrodynamic equations and Zakharov equations are derived by moment equation and two time-scale methods, respectively. The conserved quantities and nonlinear entity collapse scalar law are obtained, from which we find that the conservation energy is relevant to the nonextensive parameter but momentum as well as angular momentum and the number of plasmon are not affected by the nonextensivity of system. The self-similar collapse solution of nonextensive Zakharov equations is also presented. Furthermore, we demonstrate that the nonlinear entity collapse scalar law is relevant to the nonextensive parameter and especially it allows the existence of three dimensional stable and one dimensional collapse nonlinear entity, which is significantly different from the case of Maxwellian distribution. In the extensive limit, all the results obtained in Maxwellian framework are reproduced.

1. Introduction

Nonextensive statistical mechanics (NSM) induced by a quantity normally scaled in multifractals has been developed recently as a very useful tool to describe the complex systems whose properties cannot be exactly described by Boltzmann–Gibbs statistical mechanics [1]. For nonextensive parameter $q < 1$, it gives the distribution having suprathermal power-law tails at high energies [2], which have been proved to be equivalent to kappa distribution [3]; for $q > 1$, it derives

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the distribution suitable for the description of systems containing only low-speed particles and when the parameter $q \rightarrow 1$ the Maxwellian distribution is recovered [4]. NSM has been successfully applied to various fields [5], for instance in biological systems [6], plasma [7–9], dissipative optical lattices [10,11], spin-glasses [12], trapped ions [13], high-energy collisions at LHC/CERN (CMS, ALICE, ATLAS and LHCb detectors) and at RHIC/Brookhaven (PHENIX detector) [14–23], granular matter [24], neutrino mixing [25–29], low-dimensional maps, e.g. the (area-preserving) standard map [30] (see Bibliography in http://tsallis.cat.cbpf.br/biblio.htm). In particular, Liu et al. [31] showed a reasonable indication for the non-Maxwellian velocity distribution of plasma experimentally.

Hydrodynamic equations as the important tool to describe the real world are used widely in various fields, e.g., cosmology [32], plasma physics [33], particle physics [34], and so on. However, a large number of physical events in the real world conform to the nonextensive statistics, thus generalizing the hydrodynamic equations into the nonextensive regime is necessary. The Zakharov equations are important tool for the analysis of weakly nonlinear waves and have been used for the discussion of wave instabilities and hence become convenient starting point for the study of collapsing dynamics [35]. Plasma is a complex system and it is relevant to nonextensive effects [36] which is appearing in numerous aspects such as conserved quantities and collapsing scale law.

Herein we introduce nonextensive generalizations of the two important equations of collapsing dynamics of plasmons, namely, the hydrodynamic and Zakharov equations. The present proposals consist in deriving Maxwellian hydrodynamic and Zakharov equations into nonextensive ones, as done for the hydrodynamic and Zakharov equations in Boltzmann–Gibbs statistics. An interesting aspect about these generalizations is that nonextensive Zakharov equations allow the existence of three dimensional stable and one dimensional collapse nonlinear entities which is significantly different from the previous Maxwellian case, but coincides with the natural phenomena.

2. Nonextensive hydrodynamic equations

Let us first consider ideal fluid only existing plasma pressure and electromagnetic force tending to be isotropic by taking the frequent collisions of particles in the system into account, so the following equations are appropriate:

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{v}_s) = 0, \tag{1}$$

$$m_s n_s \left(\frac{\partial}{\partial t} + \mathbf{v}_s \cdot \nabla \right) \mathbf{v}_s = n_s \left(e_s \mathbf{E} + \frac{e_s}{c} \mathbf{v}_s \times \mathbf{B} \right) - \nabla P_s + \mathbf{q}_s, \tag{2}$$

where n, $\mathbf{v}$, m, $\mathbf{E}$, c, $\mathbf{B}$, P and $\mathbf{q}$ are the number density, velocity, mass, electric field speed of light in vacuum, magnetic field, thermal pressure, and the momentum variability, respectively. Subscript s denotes the kinds of particles. These equations above are obtained by kinetic moment methods. Let us remind the reader that when the nonextensivity of plasma is considered, the term $-\nabla P_s$ in Eq. (2) is new and different from those of previous formulations. It is relevant to the polytropic process

$$P_s = \kappa_s n_s^{\gamma_s}, \tag{3}$$

where κ_s is constant, and γ_s is polytropic index that relevant to the nonextensivity. For electron case ($s = e$), γ_e is electron specific heat ratio which is a dynamic coefficient that cannot be given in the theory of two-fluid. However, for the longitudinal oscillations $\left(\nabla \times \mathbf{v}_f^e = 0\right)$, omitting the nonlinear term in the right-hand of equation [see p. 133, Eq. (5.39) of [35] or Eq. (2.18) of [37]] $\nabla \times \nabla \times \mathbf{v}_f^e + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{v}_f^e + \frac{1}{c^2}\omega_{pe}^2\mathbf{v}_f^e - \frac{\gamma_e v_{Te}^2}{c^2}\nabla\left(\nabla \cdot \mathbf{v}_f^e\right) = -\frac{1}{c^2}\omega_{pe}^2\frac{\delta n}{n_0}\mathbf{v}_f^e$, where $\mathbf{v}_f^e$ is the fast-time-scale velocity field of electrons, v_{Te} notes the electron thermal velocity, we obtain the linear dispersion law $\omega^2 = \omega_{pe}^2 + \gamma_e k^2 v_{Te}^2$, which comparing with the Langmuir dispersion relationship in the context of nonextensive statistics [38] $\omega^2 = \omega_{pe}^2 + \frac{6}{3q_e-1}k^2 v_{Te}^2$ gives $\gamma_e = \frac{6}{3q_e-1}$. The introduction of Eq. (3) is called truncation approximation, whose actual content is using the polytropic process to approximately instead the second moment equation, in order to achieve truncated, thus closing the MHD equations. According to the method of moment equation, ∇P_s should be $\nabla \cdot \vec{P}_s$, and this approximation is based on the consideration that collisions always tends to eliminate anisotropic.

3. Nonextensive Zakharov equations

We will now focus on nonextensive Zakharov equations for which the above nonextensive hydrodynamic equations are based on. It should be emphasized at this point that when take into account the complex interaction in plasma, the nonextensive effect is need to be considered inevitability, which reflects in nonextensive parameter q. Through the following dimensionless variables: $\mathbf{r}' = \frac{2\sqrt{\mu}}{3}k_d\mathbf{r}$, $t' = \frac{2\mu}{3}\omega_{pe}t$, $\mathbf{E}'\left(\mathbf{r}', t'\right) = \frac{\sqrt{3}\mathbf{E}(\mathbf{r},t)}{8(\mu T_e \pi n_0)^{1/2}}$, $n = \frac{3}{4\mu}\frac{\delta n}{n_0}$, $\alpha = \frac{c^2}{3v_{Te}^2}$, $\mu = \frac{m_e}{m_i}$, and omitting the apostrophe, we introduce the following new nonextensive Zakharov equations (see Ref. [33] for previous different formulations)

$$i\frac{\partial}{\partial t}\mathbf{E} + \alpha\nabla \times \nabla \times \mathbf{E} - \frac{2}{3q_e - 1}\nabla\left[\nabla \cdot \mathbf{E}\right] + n\mathbf{E} = 0, \tag{4}$$