Physica A 507 (2018) 175–191

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Analytical valuation for geometric Asian options in illiquid markets

Zhe Li, Wei-Guo Zhang *, Yong-Jun Liu *

School of Business Administration, South China University of Technology, Guangzhou 510641, China

HIGHLIGHTS

- The pricing problems of Asian options in illiquid markets are investigated.
- We derived the closed-form solutions for the prices of geometric Asian options.
- We presented the put–call parity relations for the geometric Asian options.
- Numerical results show that market liquidity has an importance role in option pricing.

ARTICLE INFO

Article history:
Received 27 January 2018
Received in revised form 14 April 2018

Keywords:
Asian options
Option pricing
Market liquidity
Partial differential equations
Monte Carlo simulation

ABSTRACT

Classical option pricing theories are usually built on the paradigm of competitive and frictionless markets, while ignoring the impact of market liquidity on asset prices. In this paper, we study the pricing problems of the continuously monitored fixed and floating strike Asian options in imperfectly liquid markets. First, we assume that the dynamics of the underlying asset price follow a liquidity-adjusted Black–Scholes model. And then we derive the partial differential equation for the continuous fixed and floating strike geometric Asian options based on the delta-hedging strategy. Meanwhile, we also present the PDE governing the price of the continuous fixed and floating strike arithmetic Asian options. Second, the analytical pricing formulas for the continuous fixed and floating strike geometric Asian call options are derived by using the PDE method. Besides, we demonstrate the put–call parity relations for the continuous fixed and floating strike geometric Asian options. Finally, numerical experiments are performed to illustrate the accuracy and efficiency of the proposed liquidity-adjusted option pricing model through comparing the analytical solution with Monte Carlo simulation. Furthermore, we investigate the sensitivity of the continuous arithmetic and geometric Asian option prices to the liquidity factors. The numerical results support our idea of introducing market liquidity effect into option pricing framework.

1. Introduction

Asian options are contingent claims whose payoffs are determined by some average of the underlying asset price process over a specified time period and the strike price. The average of the underlying asset prices is observed with continuous or discrete sampling and the strike price is determined by fixed or floating value. Generally speaking, Asian options can be divided into two types: the geometric average Asian option and the arithmetic average Asian option. Since the average

* Corresponding authors.
E-mail addresses: bmzheli@mail.scut.edu.cn (Z. Li), wgzhang@scut.edu.cn (W.-G. Zhang), bmyjliu@scut.edu.cn (Y.-J. Liu).

https://doi.org/10.1016/j.physa.2018.05.069
0378-4371/

price of the underlying asset has a lower volatility so that the price of Asian option is cheaper than a plain vanilla option on the same underlying asset. Nowadays, Asian options are actively traded on equity, currency, interest rate, commodity and energy markets and are particularly employed by investors or traders for hedging transactions whose cost is related to the average of the underlying asset prices. Therefore, Asian options are the most popular path-dependent options in financial industry.

So far there have been many valuation methods for Asian options, which include various quasianalytical approaches, numerical approximation, finite difference approach, Monte Carlo simulation. For the geometric Asian option, the analytical and explicit solutions have been derived by Kemna and Vorst [1], Vorst [2] and Angus [3] in the framework of Black and Scholes [4] economy. However, many empirical observations investigate that stochastic volatility models are more suitable alternatives to capture the implied volatility smile and fattailed distribution of the underlying asset price return. In view of this phenomenon, Wong and Cheung [5] considered a fast mean-reverting stochastic volatility model and derived a semi-analytical pricing formulas for the continuously monitored geometric Asian options by means of a perturbation method. Afterwards, Kim and Wee [6] derived the explicit solutions for the prices of the continuously monitored geometric Asian options under the assumption that the underlying asset prices followed Heston's stochastic volatility model. Considering that the underlying asset price processes exhibited both stochastic volatility and jumps, Hubalek and Sgarra [7] and Hubalek et al. [8] provided semi-explicit valuation formulas for the geometric Asian options under continuous monitoring sampling. For more related works, one can refer to Fusai and Meucci [9], Zhang and Oosterlee [10], Cai et al. [11], Lee [12], Cui et al. [13], Zhang et al. [14] and among others.

The previous literature mentioned above considers geometric Asian options under the idealized assumptions, i.e. the market is frictionless and the underlying assets are perfectly liquid. However, a large number of empirical studies indicate that the liquidity is an important and significant effect factor in many financial asset prices. It has been shown that the stock returns are influenced by liquidity risk and commonality in liquidity, and the expected returns are positive related to the level of stock illiquidity. Therefore, market liquidity has currently become an issue of high concern in asset pricing and financial risk management. Naturally, investors demand a premium for bearing the risk of illiquidity. The illiquidity premium was first documented for the stock market in [15].[1] Soon afterwards, some researches began to focus on the effects of liquidity on option prices. By using a unique dataset that options are issued by a central bank and are not traded prior to maturity, Brenner et al. [29] examined the effect of illiquidity on the price of currency options. Chou et al. [30] illustrated the influence of both spot and option liquidity levels on option prices. Nordén and Xu [31] investigated the dynamic relationship between the steepness of the volatility smirk and relative index option liquidity. Christoffersen et al. [32] presented a significant positive impact of option illiquidity on expected call option returns by utilizing portfolio sorts and cross-sectional regressions. Theoretically, stock illiquidity is positively related to option illiquidity and thus indirectly affects option prices.

There have been some theoretical researches on how to introduce liquidity effect into the framework of European option pricing. Cetin et al. [33] developed a model for the inclusion of liquidity risk into arbitrage pricing theory by assuming that a stochastic supply curve for a security's price as a function of trade size.[2] Bakstein and Howison [36] developed a parameterized model for liquidity effects arising from the traded assets, where the liquidity was defined by means of a combination of a trader's individual transaction cost and a price slippage impact. Based on this model, options can be priced and hedged under the risk-neutral world. In contrast to the standard framework and consistent with a market with imperfect liquidity, Liu and Yong [37] examined the effect of stock liquidity on the replication of a European contingent claim under the assumption that the investors's trading had a direct impact on the stock price. As the liquidity was understood as a nonlinear transaction cost incurred as a function of rate of change of portfolio, Rogers and Singh [38] proposed a model for the effects of illiquidity and explored some of its consequences for the hedging of European options in the model of Black and Scholes [4]. Working within a Markovian regime-switching setting, Ludkovski and Shen [39] defined illiquidity as the inability to trade in a timely way and then studied the pricing and hedging problems of European options with liquidity shocks. On the basis of the price dynamics proposed by Brunetti and Caldarera [22], Feng et al. [40,41] derived the corresponding pricing formulas for European options and investigated the importance of the stock liquidity on option pricing. Their empirical results provided strong evidence to support that incorporating a stock liquidity into the framework of option pricing can produce smaller pricing errors. Within the framework of conic finance, Leippold and Scharer [42] studied the discrete time option pricing with stochastic liquidity model, in which the liquidity measure was not directly defined by observable variables in the financial market such as bid–ask spreads or trading volume, but inferred from a comparison of market and model implied bid–ask spreads. Recently, Li et al. [43] studied the European quanto option pricing in the presence of liquidity risk and empirical results investigated the importance of market liquidity on European quanto option prices. To the best of our knowledge, until now there is no literature research on pricing geometric Asian options in imperfectly liquid market. So the aim of this paper is to fill this gap.

Motivated by the above mentioned insights, we study the pricing problems of the continuously monitored fixed and floating strike Asian options in this paper. First, we propose to price continuous Asian options when the underlying asset price processes are driven by a Brunetti and Caldarera [22] model, which contains the Black–Scholes economy as the limiting case

1 For more related studies on illiquidity premia in the stock market, see [16–28].

2 Following the framework of Cetin et al. [33], Cetin et al. [34,35] studied the problems of option pricing and super-replication in an extended Black–Scholes economy, where the underlying asset is imperfectly liquid.