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New Bayesian frequency–magnitude distribution model for earthquakes applied in Chile

Ewin Sánchez C. [a,b,*], Pedro Vega-Jorquera [a]

[a] Departmento de Física y Astronomía, Facultad de Ciencias, Universidad de La Serena, La Serena, Chile
[b] Universidad Tecnológica de Chile INACAP, Chile

HIGHLIGHTS

- Seismic frequencies–magnitudes follow a distribution connected to the superstatistics form.
- It is possible to obtain, with a new Frequency–magnitude model, an alternative form for the b Gutenberg–Richter parameter.
- It is possible to obtain, with a new Frequency–magnitude model, excellent goodness-of-fit to samples data from Chilean earthquakes.

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ABSTRACT

We outline a model developed from a Bayesian approach, which can be connected, from the perspective of Mathai's pathway model, with the superstatistics representation presented by Beck and Cohen in 2002, which has been widely applied in non-equilibrium and complex systems. We evaluated goodness-of-fit to the observed frequency–magnitude distribution before and after major earthquakes occurred in Chile, from 2010 year to date. This new model, allows us to obtain an alternative Gutenberg–Richter b parameter, through a wider range of magnitudes.

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1. Introduction

Since the proposal submitted by Gutenberg and Richter in [1], through the empirical formula:

$$\log N = a - bm \tag{1}$$

which provides the number of earthquakes N, with magnitude $\geq m$, observed in certain region over a certain period of time, (a and b are positive parameters), many efforts have been made to find a more complete model.

In recent years, Sotolongo-Costa and A. Posadas [2], through fragment size distribution (produced by breakage during tectonic activities) and the Tsallis statistics, also proposed a relation for frequency–magnitude distribution. Subsequently other authors, as in [3], have produced new models based on the original works of Sotolongo-Costa and A. Posadas. In addition, several applications have been made through that perspective, such as [4] and [5], where non-extensive properties for behavior and evolution of seismic systems are addressed in different regions of our planet.

We derived our model using a Bayesian approach, and this, from the mathematical statistics point of view, can be connected to the Beck–Cohen formalism (which they named superstatistics [6]), through the Mathai's pathway model [7]. In Section 2, we present the fundamental Bayesian framework, and the step-by-step model construction we followed, as well as posterior goodness-of-fit tests with respect to experimental data. In Section 3, we discuss the implications, analysis and conclusions.

* Corresponding author at: Departmento de Física y Astronomía, Facultad de Ciencias, Universidad de La Serena, La Serena, Chile.
E-mail address: esanchez@userena.cl (E. Sánchez C.).

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2. Fit model

2.1. Bayesian framework

Marginalization is a very important technique applied in the Bayesian inference process, by use of which we may obtain an unconditional model for a random observable variable X, which depends of a parameter Θ.

Marginalization can be realized assuming Θ itself as a random variable, which can take values $\Theta = \{\theta_0, \theta_1, \ldots, \theta_m\}$, with the probability distribution given by a function $P(\Theta)$.

Each of the joint probabilities we know are given by the product rule,

$$P(x_i, \theta_j) = P(x_i|\theta_j)P(\theta_j) \tag{2}$$

Thus, in general, for X and Θ continuous random variables, and given our knowledge I of the system, marginal distribution $P(X)$ is given by

$$P(X) = \int P(X|\Theta, I)P(\Theta|I)\,d\Theta \tag{3}$$

This marginalization incorporates the uncertainty associated with assigning a specific value to the Θ parameter, as well as the intrinsic uncertainty present in the observed system's properties.

2.2. Model construction

Take a geographic region that has some seismic activity, where the magnitude m is a scalar variable related to the energy released in each seismic event.

We use an expression that is compatible with the Gutenberg–Richter relation of magnitudes (pertinence can be seen in [8]), and assume that it is suitable for the considered geographic region,

$$f(m) = e^{-\beta(m-m_0)} \tag{4}$$

where m_0 is the minimum threshold for the magnitudes considered. The β parameter must take a positive value, greater than zero, and its relation to the Gutenberg–Richter parameter b is:

$$b = \frac{\beta}{\ln 10} \tag{5}$$

This parameter b, of the Gutenberg–Richter magnitude–frequency relationship, has been the subject of many studies, since it is associated with the intensity of the seismic activity observed in the region of interest.

We want to determine, by marginalization, the unconditional frequency distribution of magnitudes m, considering the information I we have about this type of system.

Note that if the system information is given by I_0, then

$$P(m|I_0) = \langle \delta\big(m(\overrightarrow{x}) - m\big)\rangle_{I_0} \tag{6}$$

where $\overrightarrow{x}$ is the vector of all physical parameters hidden at the time we observe the system, which are involved in the manifestation of observed seismic activity. By not knowing all of them, makes it impossible for us to accurately predict the values of m each time an event occurs. It could be parameters that account for the physical process of elastic strain accumulation and the triggering mechanism, or other processes involved in mechanical stresses due to plate movements (e.g. [9,10]). Thus,

$$P(m|I_0) = f\big(m(\overrightarrow{x})\big)\int d\overrightarrow{x}\,\delta\big(m(\overrightarrow{x}) - m\big) \tag{7}$$

that is, $P(m|I_0) = f\big(m(\overrightarrow{x})\big)\,\Omega(m)$, where $\Omega(m)$ must be the density of states.

Since (4) has a similar form to the Boltzmann distribution (as it can also be seen in [11] within a context of earthquake statistics), and also, the magnitude m is related to the release of energy, then we propose $\Omega \sim (m - m_0)^{\frac{3n}{2}-1}$ to obtain a normalized conditional density function of m given β, so we have

$$p(m|\beta) = \frac{e^{-\beta(m-m_0)}(m - m_0)^{\frac{3}{2}n-1}\beta^{\frac{3}{2}n}}{\Gamma(\frac{3}{2}n)} \tag{8}$$

where n is related to the system's degree of freedom. On the other hand, we hypothesize that β has a gamma distribution (relevance of the chosen distribution can be seen in [12,13]). Thus,

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