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A new weighted (α, β)-norm information measure with application in coding theory

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HIGHLIGHTS

- A new weighted (α, β)-norm information measure.
- A new weighted directed divergence measure based on proposed information measure.
- Application of proposed information measure in coding theory.

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ABSTRACT

In the present communication, we introduce a quantity which is called weighted (α, β)-norm entropy and discuss its some major properties with Shannon and other entropies in the literature. Corresponding to the proposed entropy, a new weighted directed divergence measure has been introduced and its validity is established. Further, we give the application of (α, β)-norm entropy in coding theory and a coding theorem analogous to the ordinary coding theorem for a noiseless channel has been proved. The theorem states that the proposed entropy is the lower bound of mean code word length.

1. Introduction

If the signals transmitted through a communication system are considered as random abstract events, the qualitative aspect of the information developed is based upon the probabilities of occurrence of these events. For such a discrete system, an information scheme may be represented in the form

$$I = \begin{bmatrix} E_1, & E_2, \ldots, E_n \\ a_1, & a_2, \ldots, a_n \end{bmatrix} = \begin{bmatrix} E \\ A \end{bmatrix}, \tag{1.1}$$

where $E = (E_1, E_2, \ldots, E_n)$ represents a family of events corresponding to random experiment and $A = (a_1, a_2, \ldots, a_n)$, $a_i \geq 0$, $\sum_{i=1}^{n} a_i = 1$ is the probability distribution defined over it. Shannon information measure [1] for the scheme (1.1) is

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given by

$$H(A) = -\sum_{i=1}^{n} a_i \log(a_i). \tag{1.2}$$

The quantity, in some sense, measure the amount of information contained in the scheme. However, this measure does not take into account the effectiveness (or importance) of the events involved, with respect to observer goal pair. This is so because (1.2) depends only on the probabilities associated with the events of the information scheme. In a practical situation of probabilistic nature, there are quite often subjective considerations getting involved with the study. These considerations take into account the effectiveness of the outcomes. Inspired by this idea, Belis and Guiasu [2] introduced a 'utility distribution' $U = (u_1, u_2, \ldots, u_n)$, where each u_i is a non-negative real number accounting for the utility of occurrence of the ith event, with the probability scheme A.

As a consequence, studies may be based on what may be called 'Utility Information Scheme' given by

$$I^\star = \begin{bmatrix} E_1, & E_2, \ldots, E_n \\ u_1, & u_2, \ldots, u_n \\ a_1, & a_2, \ldots, a_n \end{bmatrix} = \begin{bmatrix} E \\ U \\ A \end{bmatrix}. \tag{1.3}$$

It is worthwhile to mention here that the utility u_i of an event is in general independent of its probability of occurrence a_i. Belis and Guiasu [2] introduce the function

$$H(A, U) = -\sum_{i=1}^{n} u_i a_i \log(a_i), \tag{1.4}$$

where $u_i > 0$ and $\sum_{i=1}^{n} a_i = 1$ which they considered as a satisfactory measure for the average number quantity of valuable or useful information provided by a source letter.

It may be observed that $H(A, U)$ reduces to (1.2) when utilities are not taken into account, *i.e.*, each $u_i = 1$.

Measure (1.1) is additive in nature,

$$i.e., \qquad H(A \star B, U \star V) = \bar{V} H(A; U) + \bar{U} H(B; V). \tag{1.5}$$

where $B = (b_1, b_2, \ldots, b_n)$, $b_j \geq 0$, $\sum_{j=1}^{n} b_j = 1$ and $V = (v_1, v_2, \ldots, v_n)$, $v_j > 0$, is the utility distribution associated with B, $\bar{U} = \sum_{i=1}^{n} a_i u_i$, $\bar{V} = \sum_{j=1}^{n} b_j v_j$,

$$U \star V = \{u_i v_j : u_i \in U, v_j \in V\},$$
$$A \star B = \{a_i b_j : a_i \in A, b_j \in B\}.$$

In this paper, we extend the idea of Boakee and Lubbe entropy [3] and Shannon entropy [4] and propose a new weighted (α, β)-norm information measure and (α, β)-norm divergence measure. Further, studying its properties and giving its applications in information theory, this paper is organized as follows:

The section 'Introduction' presents the work done by the previous researchers in the field. A new weighted (α, β)-norm information measure is proposed in Section 2. Further, the relation between the proposed information measure and some existing measures is also established in this Section. Validity of proposed weighted information measure is established in Section 3. Corresponding to the proposed information in Section 2, a new weighted directed divergence measure is proposed and validated in Section 4. Section 5 is devoted to the application of proposed weighted information measure in coding theory. At last, paper is concluded with 'Conclusions' in Section 6.

2. A new weighted (α, β)-norm information measure

Now, we introduce a new weighted information measure as:

$$H_\alpha^\beta(A; U) = \frac{\alpha \times \beta}{\alpha - \beta} \left[\left(\sum_{i=1}^{n} \frac{u_i a_i^\beta}{\sum_{i=1}^{n} (u_i a_i)} \right)^{\frac{1}{\beta}} - \left(\sum_{i=1}^{n} \frac{u_i a_i^\alpha}{\sum_{i=1}^{n} (u_i a_i)} \right)^{\frac{1}{\alpha}} \right], \tag{2.1}$$

where either $0 < \alpha < 1$; $1 < \beta < \infty$ or $0 < \beta < 1$; $1 < \alpha < \infty$. Since the proposed information measure is symmetrical, therefore, throughout this manuscript we will consider $0 < \alpha < 1$, $1 < \beta < \infty$ only.

Particular Cases:

1. If $u_i = 1$, then (2.1) becomes

$$H_\alpha^\beta(A) = \frac{\alpha \times \beta}{\alpha - \beta} \left[\left(\sum_{i=1}^{n} a_i^\beta \right)^{\frac{1}{\beta}} - \left(\sum_{i=1}^{n} a_i^\alpha \right)^{\frac{1}{\alpha}} \right], \tag{2.2}$$

which is an (α, β)-norm entropy studied by Joshi and Kumar [5].