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Braess paradox in a network with stochastic dynamics and fixed strategies

Stefan Bittihn *, Andreas Schadschneider

Institute for Theoretical Physics, University of Cologne, 50937 Köln, Germany

HIGHLIGHTS

- Braess' paradox appears in networks of exclusion processes.
- Phase diagram is determined for drivers with fixed strategies in Braess' network.
- Fixed strategies lead to a disappearance of fluctuating domain walls.
- Gridlocks are possible in parts of the phase diagram.
- The mostly negative effects of the new road for the network users are quantified.

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ABSTRACT

The Braess paradox can be observed in road networks used by selfish users. It describes the counterintuitive situation in which adding a new, per se faster, origin–destination connection to a road network results in increased travel times for all network users. We study the network as originally proposed by Braess but introduce microscopic particle dynamics based on the totally asymmetric exclusion process. In contrast to our previous work Bittihn and Schadschneider (2016), where routes were chosen randomly according to turning rates, here we study the case of drivers with fixed route choices. We find that travel time reduction due to the new road only happens at really low densities and Braess' paradox dominates the largest part of the phase diagram. Furthermore, the domain wall phase observed in Bittihn and Schadschneider (2016) vanishes. In the present model gridlock states are observed in a large part of phase space. We conclude that the construction of a new road can often be very critical and should be considered carefully.

1. Introduction

Urbanization is one of the big challenges of modern times. As the world population grows, more and more people are moving into cities [1]. With the growing population sizes, also the transportation network has to adapt. The expansion of the city and its transportation network, an interplay of top-down planning and self-organizational processes [2], has to be considered carefully to be efficient. The Braess paradox was discovered by D. Braess in 1968 [3,4]. He proposed a specific road network in which adding a new road counterintuitively leads to higher travel times for all the network users given that they minimize their own travel times selfishly. The network consists of four individual roads forming two possible routes from start to finish. Then a new road is added resulting in a new per se faster route from start to finish.[1] A state of the system

* Corresponding author.
E-mail addresses: bittihn@thp.uni-koeln.de (S. Bittihn), as@thp.uni-koeln.de (A. Schadschneider).

1 "Faster" in the sense of smaller travel time for a single particle compared to the original routes.

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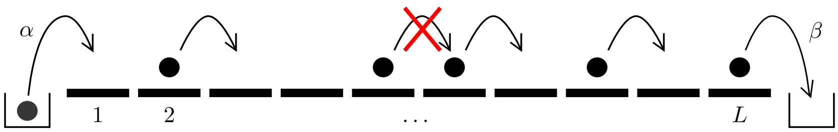

Fig. 1. A single TASEP of length L with open boundary conditions. In the case of random sequential update rules a site is chosen randomly. If it is occupied, the particle can jump to the next site if this next site is empty.

is characterized by the distribution of the vehicles onto the available routes. If a certain amount of drivers[2] wants to go from start to finish and they all want to minimize their own travel times, the system is in a stable state if all used routes have the same travel time which is shorter than the travel times of any unused routes. This is the so-called user optimum [5] or Nash equilibrium of the system. Braess showed that for specific combinations of travel time functions of the roads and the total number of cars, the user optimum of the system with the new road has higher travel times than that of the system without the new road. This appears to be a paradox since one would assume that an additional route increases the capacity and thus leads to a decrease of travel times.

Many efforts have been made in understanding Braess' paradox in more general terms. Indeed it was shown that its occurrence is very prevalent in congested networks [6]. The regions of its occurrence in certain models were determined [7,8] and it was also shown to occur in certain real-world scenarios [9]. Furthermore, analogues of the paradox were e.g. found in mechanical networks [10], energy networks [11], pedestrian dynamics [12] and thermodynamic systems [13]. In most studies on the paradox the focus was on macroscopical mathematical models of car traffic in which the roads are treated as uncorrelated. Travel times of the roads are given by functions which are linear in the number of cars using the roads. This lead to the discovery, a general understanding and also the observation of the effect in the real world and sparked the ongoing fascination with this counterintuitive phenomenon. Nevertheless, linear travel time functions and the fact that in those models there are no correlation effects between the roads result in a rather unrealistic description of traffic in the network.

It is important to gain a deeper understanding of the paradox in more realistic scenarios since this is relevant for the design of new roads in real road networks. Especially the effects of a more realistic microscopic dynamics and also inter-road correlations like jamming effects or conflicts at road junctions need to be understood. In a recent article [14] we have studied Braess' network where the dynamics on the edges is modelled by totally asymmetric exclusion processes (TASEPs). The TASEP is a simple cellular automaton [15] and is now renowned as the paradigmatic model for single-lane traffic. It covers a lot of effects which are not included in deterministic mathematical models. In [14] we studied the case where all drivers are identical and choose their routes stochastically. The route choice is determined through turning probabilities on the junction sites. We found that the paradox occurs at intermediate global densities $0.1 \lesssim \rho_{\text{global}} \lesssim 0.3$. A large part of the phase diagram is dominated by the so-called fluctuation-dominated regime in which no stable travel times can be measured due to domain walls of fluctuating positions (i.e. traffic jams of fluctuating lengths).

In the present paper we analyse the same network with TASEP dynamics on the edges. Instead of a random route choice based on turning probabilities at junctions the particles have fixed routes. This corresponds e.g. to the scenario of daily commuters who stick to their 'favourite' routes. In the present model stable travel times can be found throughout the whole phase diagram. The fluctuation-dominated regime which is a defining phase for the model we studied previously disappears. Braess' paradox is found to be even more prevalent in the present case as the system shows Braess-like behaviour in almost the whole phase space for densities $\rho_{\text{global}} \gtrsim 0.1$. This implies that the paradox is even more prominent and important in networks of microscopic transport models than we concluded due to our findings in [14].

2. Model definition

2.1. The totally asymmetric exclusion process

The TASEP is a one-dimensional paradigmatic stochastic transport model. A single TASEP consists of L cells (L is also called the length of a TASEP) which can be either empty or occupied by one particle. In our analysis we use the so-called random sequential update rules. With this update scheme, the dynamics works as follows: With uniform probability one of the L cells is chosen. If this cell is occupied by a particle, the particle can jump to the next cell iff the next cell is empty (see Fig. 1). After L of those updates, one timestep is completed.[3] In the case of open boundary conditions, particles are fed onto site 1 from a reservoir which is occupied with the entrance-probability α. Particles can leave the system from site L with the exit probability β. In the case of periodic boundary conditions site $L+1$ is identified with site 1. Thus the total number of particles M in the system is constant and the system effectively becomes a ring.

[2] Throughout this article we use the terminology "driver", "vehicle", "particle" synonymously.

[3] Note that not necessarily all particles are updated in a timestep and some particles can be updated more than once.