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Asymptotic behavior for a long-range Domany–Kinzel model

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HIGHLIGHTS

- We consider a long-range Domany–Kinzel model for directed bond percolation on the rectangular lattice.
- The critical aspect ratio for this percolation model in the thermodynamic limit is corrected.
- The asymptotic behavior of this percolation model near the critical aspect ratio is obtained.
- We investigate the cases with infinite bonds from one vertex to the next row for this model.
- A special case with infinite bonds is studied in details, and its critical power is derived.

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ABSTRACT

We consider a long-range Domany–Kinzel model proposed by Li and Zhang (1983), such that for every site (i, j) in a two-dimensional rectangular lattice there is a directed bond present from site (i, j) to $(i+1, j)$ with probability one. There are also $m+1$ directed bounds present from (i, j) to $(i-k+1, j+1)$, $k = 0, 1, \ldots, m$ with probability $p_k \in [0, 1)$, where m is a non-negative integer. Let $\tau_m(M, N)$ be the probability that there is at least one connected-directed path of occupied edges from $(0, 0)$ to (M, N). Defining the aspect ratio $\alpha = M/N$, we derive the correct critical value $\alpha_{m,c} \in \mathbb{R}$ such that as $N \to \infty$, $\tau_m(M, N)$ converges to 1, 0 and 1/2 for $\alpha > \alpha_{m,c}$, $\alpha < \alpha_{m,c}$ and $\alpha = \alpha_{m,c}$, respectively, and we study the rate of convergence. Furthermore, we investigate the cases in the infinite m limit. Specifically, we discuss in details the case such that $p_n \in [0, 1)$ with $n \in \mathbb{Z}_+$ and $p_n \approx_{n\to\infty} pn^{-s}$ for $p \in (0, 1)$ and $s > 0$. We find that the behavior of $\lim_{m\to\infty} \tau_m(M, N)$ for this case highly depends on the value of s and how fast one approaches to the critical aspect ratio. The present study corrects and extends the results given in Li and Zhang (1983).

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1. Introduction

Directed percolation, or oriented percolation, can be thought of simply as a percolation process on a directed lattice in which connections are allowed only in a preferred direction. It was first studied by Broadbent and Hammersley in 1957 [1], and it has remained to this day as one of the most outstanding interesting problems in probability and statistical mechanics. Furthermore, directed percolation is closely related to the Reggeon field theory in high-energy physics and the Markov processes with branching, recombination and absorption that occur in chemistry and biology [2,3], etc. Various properties, results and conjectures of directed percolation can be found in [4,5] and the references therein. However, very little is known in the way of exact solutions for the directed percolation problem.

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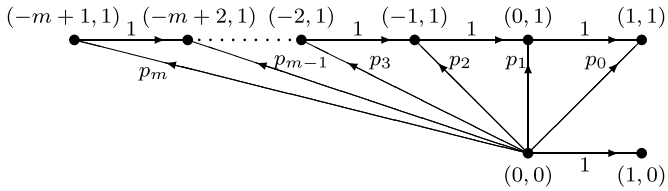

Fig. 1. The long-range directed bond percolation model.

Domany and Kinzel [6] defined a solvable version of compact directed percolation on the square lattice in 1981 as follows. Given a fixed $p \in (0, 1)$, each vertical bond is directed upward with occupation probability p (independently of the other bonds), while each horizontal bond is directed rightward with occupation probability 1. Furthermore, it is known that the boundary of the Domany–Kinzel model has the same distribution as the one-dimensional last passage percolation model [7]. A three-dimensional version of Domany–Kinzel model with occupation probability 1 along two spatial directions was considered in Ref. [8]. Recently, the model has been considered in more general cases. For example, the directed percolation model on the square lattice whose vertical edges occupied with a probability p_v and horizontal edges in the nth row occupied with a probability 1 if n is even and p_h if n is odd [9]; the directed percolation model on the triangular lattice in terms of a square lattice with vertical probability y, horizontal probabilities 1 and x alternatively, and diagonal edges from lower-left to upper-right or from lower-right to upper-left with probability d [10]; the directed percolation model on the honeycomb lattice as bricks such that vertical edges are directed upward with probability y, and horizontal edges are directed rightward with probabilities 1 and x in alternate rows [11]; the directed percolation model on square lattice such that horizontal edges are directed rightward with probabilities one, and vertical edges are directed upward with probabilities p_1, p_2 alternatively in even rows and probabilities p_2, p_1 alternatively in odd rows [12].

In this article, we consider the long-range directed bond percolation models on the square lattice as follows. The horizontal edges are directed rightward with probabilities one, and the directed bond from (i, j) to $(i-k+1, j+1)$, $k = 0, 1, \ldots, m$ with respective probabilities $p_k \in (0, 1)$, where m is a non-negative integer (see Fig. 1). Notice that this model has been considered in [13]. However, the critical value derived there is questionable, and the rate of convergence has not yet been discussed.

The vertices (sites) of the square lattice are located at a two-dimensional rectangular net $\{(i, j) \in \mathbb{Z} \times \mathbb{Z}_+ : i \le M$ and $0 \le j \le N\}$. Throughout this article, we denote $q_k = 1 - p_k$, $k = 0, 1, \ldots, m$ for convenience. We say that the vertex (i, j) is percolating if there is at least one connected-directed path of occupied edges from $(0, 0)$ to (i, j). Given any $\alpha \in \mathbb{R}$, denote $N_\alpha = \lfloor \alpha N \rfloor = \sup\{i \in \mathbb{Z} : i \le \alpha N\}$ with $N \in \mathbb{Z}_+$. Let $\mathbb{P}$ be the probability distribution of the bond variables, and define the two point correlation function

$$\tau_m(N_\alpha, N) = \mathbb{P}((N_\alpha, N) \text{ is percolating}) . \tag{1.1}$$

2. Main results

For convenience, define

$$\bar{q}_m = q_0 \cdots q_{m-1} q_m ,$$

and the notation $a_1 \approx a_2$ means that $a_1/a_2 \in (0, \infty)$. $f(N) \approx_{N\to\infty} g(N)$ and $f(\alpha) \approx_{\alpha\to\alpha_c} g(\alpha)$ mean that the limits $\lim_{N\to\infty} f(N)/g(N)$ and $\lim_{\alpha\to\alpha_c} f(\alpha)/g(\alpha)$ are bounded from zero and infinite, respectively. We have the following main theorem.

Theorem 2.1. *Given a finite $m \in \mathbb{Z}_+$ and $p_k \in (0, 1)$, $k = 0, 1, 2, \ldots, m$, there is a critical aspect ratio*

$$\alpha_{m,c} = \frac{\prod_{j=1}^m q_j^j}{1 - \bar{q}_m} - \sum_{k=2}^m \left(1 - q_k q_{k+1}^2 \cdots q_m^{m-k+1}\right) , \tag{2.1}$$

such that

$$\tau_m(N_{\alpha_{m,c}}, N) = \frac{1}{2} + O\left(\frac{1}{\sqrt{N}}\right) \tag{2.2}$$

in the large N limit, and when α is close to $\alpha_{m,c}$ but not equal to $\alpha_{m,c}$

$$\begin{aligned} \tau_m(N_\alpha, N) &\le e^{-NI(\alpha)} \quad \textit{for} \quad \alpha < \alpha_{m,c} , \\ 1 - \tau_m(N_\alpha, N) &\le e^{-NI(\alpha)} \quad \textit{for} \quad \alpha > \alpha_{m,c} , \end{aligned}$$

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