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Global stability in a mathematical model of de-radicalization

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HIGHLIGHTS

- De-radicalization modeled with an epidemic model.
- Five compartments are considered: Susceptible, Recruiters, Extremists and Treatment.
- The dynamics is determined by the basic reproduction number $\mathcal{R}_0$.
- If $\mathcal{R}_0 > 1$ the equilibrium with no terrorists is globally stable, and extremists and recruiters head for extinction.
- Model is used to assess strategies to counter violent extremism.

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ABSTRACT

Radicalization is the process by which people come to adopt increasingly extreme political, social or religious ideologies. When radicalization leads to violence, radical thinking becomes a threat to national security. De-radicalization programs are part of an effort to combat violent extremism and terrorism. This type of initiatives attempt to alter violent extremists radical beliefs and violent behavior with the aim to reintegrate them into society. In this paper we introduce a simple compartmental model suitable to describe de-radicalization programs. The population is divided into four compartments: (S) susceptible, (E) extremists, (R) recruiters, and (T) treatment. We calculate the basic reproduction number $\mathcal{R}_0$. For $\mathcal{R}_0 < 1$ the system has one globally asymptotically stable equilibrium where no extremist or recruiters are present. For $\mathcal{R}_0 > 1$ the system has an additional equilibrium where extremists and recruiters are endemic to the population. A Lyapunov function is used to show that, for $\mathcal{R}_0 > 1$, the endemic equilibrium is globally asymptotically stable. We use numerical simulations to support our analytical results. Based on our model we assess strategies to counter violent extremism.

1. Introduction

According to Horgan [1] radicalization is the social and psychological process of incrementally experienced commitment to extremist political or religious ideology. Radicalization can lead to violent extremism and therefore it has become a major concern for national security. Typical counterterrorism strategies fall into two categories:

1. Law enforcement approach: violent extremist are investigated prosecuted and imprisoned.
2. Military approach: violent extremists are killed or captured on the battlefield.

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Practitioners of counterterrorism agree that these approaches alone cannot break the cycle of violence [2]. The realization of the inadequacy of the counterterrorism approach has lead to different strategies, collectively known as countering violent extremism (CVE). CVE is a collection of noncoercive activities whose aim is to intervene in an individual's path toward violent extremism, to interdict criminal activity and to reintegrate those convicted of criminal activity into society. CVE programs can be divided into three broad classes [2–5]

1. *Prevention programs*, which seek to prevent the radicalization process from occurring and taking hold in the first place;
2. *Disengagement programs*, which attempt to stop or control radicalization as it is occurring;
3. *De-radicalization programs*, which attempt to alter an individual extremist beliefs and violent behavior with the aim to reintegrate him into society. This type of programs often target convicted terrorists.

According to Horgan [6] there are at least 15 publicly known de-radicalization programs from Saudi Arabia to Singapore, but there are likely twice as many. In this paper we use a compartmental model to model de-radicalization programs.

The attempt to use quantitative methods in describing social dynamics is not new, and compartmental models have been used to study various aspect of social dynamics. For instance Hayward introduced a model of church growth [7], Jeffs et al. studied a model of political party growth [8], Romero et al. analyzed a model for the spread of political third parties [9] and Crisosto et al. studied the growth of cooperative learning in large communities [10]. The dynamics of the spread of crime was studied by McMillon, Simon and Morenoff [11] and by Mohammad and Roslan [12]. A mathematical model of the spread of gangs was studied by Sooknanan, Bhatt, and Comissiong [13]. The same authors studied the model for the interaction of police and gangs in [14]. Castillo-Chavez and Song analyzed the transmission dynamics of fanatic behaviors [15], Camacho studied a model of the interaction between terrorist and fanatic groups [16], Nizamani, Memon and Galam modeled public outrage and the spread of violence [17]. Compartmental models of radicalization were studied by Galam and Javarone [18] and by McCluskey and Santoprete [19].

In this paper we build on the compartmental model introduced in [19] by adding a treatment compartment. This allows us to consider de-radicalization in our analysis. We divide the population into four compartments, (S) susceptible, (E) extremists, (R) recruiters, and (T) treatment (see Fig. 1). Using this simple model, we attempt to test the effectiveness of de-radicalization programs in countering violent extremism. This is an important issue since, at least on the surface, these de-radicalization programs are promising. In fact, these programs appear to be cost effective, since they are far cheaper than indefinite detention [6]. However, the degree of government support for these programs hinges on their efficacy and, unfortunately, indicators of success and measures of effectiveness remain elusive [3].

As in [19] we use the basic reproduction number $\mathcal{R}_0$ to evaluate strategies for countering violent extremism. We will show that for $\mathcal{R}_0 < 1$ the system has a globally asymptotically stable equilibrium with no individuals in the extremist, recruiter and treatment classes, and that for $\mathcal{R}_0 > 1$ the system has an additional equilibrium in which extremists and recruiters are endemic to the population. The latter equilibrium is globally asymptotically stable for $\mathcal{R}_0 > 1$. Therefore, if $\mathcal{R}_0 < 1$ the ideology will be eradicated, that is, eventually the number of recruiters and extremists will go to zero. When $\mathcal{R}_0 > 1$ the ideology will become endemic, that is, the recruiters and extremists will establish themselves in the population. In our model the basic reproduction number is

$$\mathcal{R}_0 = \frac{\Lambda}{\mu}\frac{\beta(c_E q_E + b_E q_R - \frac{(1-k)\delta p_E}{b_T} q_R)}{b_E b_R - c_E c_R - \frac{(1-k)\delta}{b_T}(c_E p_R + b_R p_E)}, \tag{1.1}$$

where μ is the mortality rate of the susceptible population, k is the fraction of successfully de-radicalized individuals, and δ is the rate at which individuals leave the treatment compartment, so that $1/\delta$ is the average time spent in the treatment compartment. The rates at which extremists and recruiters enter the treatment compartment are p_E and p_R, respectively. Moreover, $b_E = \mu + d_E + c_E + p_E$ and $b_R = \mu + d_R + c_R + p_R$, where d_E and d_R are the additional mortality rates of the extremists and recruiters, respectively.[1] Other parameters are described in Section 2. Note that, if $p_E, p_R \to 0$, then the basic reproduction number limits to the one of the bare-bones model studied in [19].

One approach to dealing with extremism, which follows under the umbrella of counterterrorism, is to prosecute and imprison violent extremists. This approach was studied in [19] where it was shown that increasing the parameters d_E and d_R resulted in a decrease in $\mathcal{R}_0$. A similar results holds for the model studied in this paper. A different strategy consists in improving the de-radicalization programs by either increasing the success rate k or by increasing the rates p_E and p_R at which extremists and recruiters enter the T compartment. Since $\mathcal{R}_0$ is a decreasing function of k, p_E, and p_R, increasing these parameters decreases $\mathcal{R}_0$. Hence, according to our model, this is a successful strategy to counter violent extremism. Another option is to decrease δ, which in turn decreases $\mathcal{R}_0$. This approach is also viable because $\mathcal{R}_0$ is an increasing function of δ. A good way of thinking about this is to consider prison-based de-radicalization programs, in which case, decreasing δ corresponds to increasing $\frac{1}{\delta}$, the average prison sentence.

Note that, in general, it may not be easy to determine the values of parameters because available data are scarce. It has been claimed, however, that the de-radicalization program in Saudi Arabia, has a rate of recidivism of about 10–20% [6], which gives an estimate for the value of k.

[1] In the context of the present model these can be viewed as the rates at which extremists and recruiters are imprisoned with life sentences.