Physica A 506 (2018) 692–701

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Quantum Brownian motion in a magnetic field: Transition from monotonic to oscillatory behaviour

Urbashi Satpathi *, Supurna Sinha

Raman Research Institute, C. V. Raman Avenue, Sadashivanagar, Bangalore 560080, India

HIGHLIGHTS

- Brownian motion of a charged particle in a magnetic field.
- Transition from monotonic to oscillatory behaviour.
- Transition independent of memory kernel.

ARTICLE INFO

Article history:
Received 8 February 2018
Received in revised form 16 April 2018
Available online 27 April 2018

Keywords:
Quantum Langevin equation
Brownian motion
Mean square displacement
Cyclotron frequency
Dissipation
Memory kernel

ABSTRACT

We investigate the Brownian motion of a charged particle in a magnetic field. We study this in the classical and quantum domains. In both domains, we observe a qualitatively interesting transition of the mean square displacement from a monotonic to a damped oscillatory behaviour as one increases the strength of the magnetic field. We notice that these features of the mean square displacement are robust and remain essentially the same for an Ohmic dissipation model and a single relaxation time model for the memory kernel. The predictions stemming from our analysis can be tested against experiments in trapped cold ions.

1. Introduction

The problem of a Brownian particle suspended in a liquid subject to thermal fluctuations has been studied extensively [1–4]. More recently there has been work in the area of a Brownian particle undergoing diffusion driven by quantum fluctuations [5–9].

In this paper, we are interested in studying the diffusion behaviour of a charged particle in a magnetic field. There have been two approaches towards solving this problem. Leggett et al. [8,10,11] have used the Feynman Vernon path integral approach in which they have solved the dynamics of a charged particle in a magnetic field in the presence of an Ohmic bath. Subsequently, Li et al. [12,13] have approached the problem via a quantum Langevin equation which corresponds to a reduced description of the system in which the coupling with the heat bath is described by two terms: an operator valued random force $F(t)$ with mean zero and a mean force characterized by a memory function $\mu(t)$. In this paper we follow the approach of Refs. [12,13] since it is naturally suited to addressing the question of our interest: the time evolution of the mean square displacement of a charged particle in a magnetic field in the presence of viscous dissipation in the high temperature classical domain and the low temperature quantum domain.

* Corresponding author.
E-mail address: urbashi@rri.res.in (U. Satpathi).

https://doi.org/10.1016/j.physa.2018.04.085

In recent years there has been quite a lot of interest in this area [9,12–20]. In particular, in Ref. [20] the low temperature thermodynamics has been discussed in the context of dissipative diamagnetism. In contrast, our focus has been to investigate the mean square displacement growth of a charged particle in a magnetic field. Here we study in detail the interplay between the effect of the magnetic field and damping effects due to dissipation. In particular, a particle of charge q and mass m in a magnetic field B, moves in a circular orbit at a rate set by the cyclotron frequency $\omega_c = qB/mc$, where c is the speed of light. The friction coefficient γ provides a rate γ^{-1} of dissipation. We probe various different regimes of these two competing time scales both in the high temperature classical domain and the low temperature quantum domain and analyse the growth of the mean square displacement in these regimes. Furthermore, we discuss experimental implications of our theoretical results. In particular, to test our predictions experimentally one can proceed as follows [21–23]. One can consider cold atom experiments with hybrid traps [24] for ions and neutral atoms and explore the Brownian motion of a charged particle in the presence of a magnetic field induced by Helmholtz coils.

The paper is organized as follows. In Section 2 we solve the Quantum Langevin Equation for a charged particle in a viscous medium in the presence of a magnetic field [12,13]. In Section 3 we present an analytical expression for the mean square displacement. We then study the high temperature classical domain and probe two regimes — a viscosity dominated regime and a magnetic field dominated regime. We do a similar analysis in the low temperature quantum domain. We analyse the growth of the mean square displacement using two different memory kernels — the Ohmic, memory free kernel, and the single relaxation time kernel which has nontrivial memory. We find that our results are robust and independent of the details of the kernel. We finally end the paper with some concluding remarks in Section 4.

2. Quantum Langevin equation in the presence of a magnetic field

The quantum generalized Langevin equation of a charged particle in the presence of a magnetic field is given by [12,13]

$$m\ddot{\vec{r}}(t) = -\int_{-\infty}^{t} \mu(t-t')\dot{\vec{r}}(t')dt' + \frac{q}{c}(\dot{\vec{r}}(t)\times\vec{B}) + \vec{F}(t) \tag{1}$$

where, m is the mass of the particle, $\mu(t)$ is the memory kernel, q is the charge, c is the speed of light, $\vec{B}$ is the applied magnetic field and $\vec{F}(t)$ is the random force with the following properties [6]

$$\langle F_\alpha(t)\rangle = 0 \tag{2}$$

$$\frac{1}{2}\langle\{F_\alpha(t), F_\beta(0)\}\rangle = \frac{\delta_{\alpha\beta}}{2\pi}\int_{-\infty}^{\infty} d\omega \mathrm{Re}\,[\mu(\omega)]\,\hbar\omega \coth\left(\frac{\hbar\omega}{2k_BT}\right)e^{-i\omega t} \tag{3}$$

$$\langle[F_\alpha(t), F_\beta(0)]\rangle = \frac{\delta_{\alpha\beta}}{\pi}\int_{-\infty}^{\infty} d\omega \mathrm{Re}\,[\mu(\omega)]\,\hbar\omega e^{-i\omega t} \tag{4}$$

Here, $\alpha, \beta = x, y, z$, and $\delta_{\alpha\beta}$ is the Kronecker delta function, such that

$$\delta_{\alpha\beta} = \begin{cases} 1 & \text{if } \alpha = \beta \\ 0 & \text{if } \alpha \neq \beta \end{cases}$$

$\mu(\omega) = \int_0^\infty dt\mu(t)e^{i\omega t}$. Notice that $\mu(t) = 0$ for $t < 0$. This follows from causality. Eqs. (3) and (4) are obtained from the Fluctuation–Dissipation Theorem which relates the dissipative and fluctuating parts of the quantum Langevin equation (Eq. (1)). The dissipative part is characterized by the memory kernel $\mu(t)$, and the fluctuating part is characterized by the random force $\vec{F}(t)$.

We assume that the magnetic field is directed along the $z-$axis, i.e. $\vec{B} = (0, 0, B)$. Then we can write Eq. (1) in terms of components as follows:

$$m\ddot{x} = -\int_{-\infty}^{t} \mu(t-t')\dot{x}dt' + \frac{q}{c}\dot{y}B + F_x(t) \tag{5}$$

$$m\ddot{y} = -\int_{-\infty}^{t} \mu(t-t')\dot{y}dt' - \frac{q}{c}\dot{x}B + F_y(t) \tag{6}$$

The motion along the $z-$axis is the same as that of a free particle. The motion in the x–y plane is affected by the magnetic field strength. We restrict our analysis to the x–y plane and study the Brownian motion of a charged particle in a magnetic field.