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Efficient disintegration strategy in directed networks based on tabu search

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HIGHLIGHTS

- An optimization model for disintegration strategy in directed networks is established.
- An efficient disintegration strategy based on tabu search is proposed.
- This strategy is more efficient than others based on degree and betweenness.
- The critical nodes in directed networks may not have large betweenness centrality.

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ABSTRACT

The problem of network disintegration, which aims at identifying the critical nodes or edges whose removal will lead to a network collapse, has attracted much attention due to its wide applications. This paper focuses on the disintegration of directed networks. We propose a disintegration strategy based on tabu search. Experiments show that the disintegration effect of our strategy is obviously better than those of typical disintegration strategies based on local structural properties. Moreover, we find that the critical nodes identified to remove in directed networks are not those nodes with large degree or betweenness centrality that always are the crucial properties in undirected network.

1. Introduction

Complex networks describe a wide range of systems in the nature and society [1]. Examples include power grids, social networks, airline networks, urban road networks, terrorist networks, the Internet, etc. The study of complex network has become one of the most popular topics in interdisciplinary areas in the past decades. Due to its broad applications, studying on the robustness of complex networks has received growing attention and has become one of the central topics in the complex network research [2–7].

In the majority of cases, networks are beneficial, such as power grids and the Internet, whose function we want to preserve. Therefore, many studies have developed methods for enhancing the robustness of these beneficial networks [8–17]. In other situations by which this paper is motivated, we want to disintegrate some networks if they are harmful such as immunizing the population in social networks or suppressing the virus propagation in computer networks. Other examples of network disintegration include destabilizing terrorist networks [18], preventing financial contagion [19], controlling the rumor diffusion [20], and perturbing cancer networks [21]. The problem of network disintegration aims actually at identifying a set of critical nodes or edges for removal for the sake of collapsing a network, which has wide practical application in destroying network structure and controlling the information diffusion.

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Many approaches for network disintegration have been proposed in the last decades [22,23]. Early disintegration strategies were related to certain structural metrics of nodes or edges. The most classical strategy is the degree-based strategy [2]. It is shown that scale-free networks with power-law degree distributions are extremely vulnerable to intentional attacks, in which nodes are removed sequentially by the decreasing order of their degrees. Considering that the degree is a local property of the node, the betweenness centrality is introduced as a new criterion [24,25]. Other global metrics were also used to identify the vital nodes for network disintegration [26], such as coreness [27] and subgraph centrality [28]. To achieve better disintegration effect, other methods were proposed [29–33].

Most previous researches on the problem of network disintegration have been limited to undirected networks. However, most real-world networks are actually directed, such as transportation networks [34] and terrorist networks [18]. Studying on the problem of disintegration in directed networks is of great significance, but few researches concentrated on this problem, especially with the view of global optimization. In this paper, we focus on the optimal disintegration strategy in directed networks. We will introduce tabu search to identify the critical nodes. Experiments show that the disintegration effect of optimal strategy is obviously better than the strategies guided by local properties of nodes.

The paper is organized as follows. In Section 2, we present the optimization model for disintegration strategy in directed networks. In Section 3, we investigate the solution based on tabu search for the optimization model. In Section 4, the experiments in various model networks are shown. And in Section 5, we present the conclusions and discussions.

2. Optimization model for disintegration strategy in directed networks

Consider networks represented by a simple directed graph $G = (V, E)$ having no loops and multiple edges, where V is the set of nodes and E is the set of edges. Let $N = |V|$ be the number of nodes and $W = |E|$ be the number of edges, respectively. Let $A(G) = (a_{ij})_{N\times N}$ be the adjacency matrix of G, where $a_{ij} = 1$ if there is a directed edges from v_i to v_j, and $a_{ij} = 0$ otherwise. Let $k_i^- = \sum_{j=1}^{N} a_{ji}$ and $k_i^+ = \sum_{j=1}^{N} a_{ij}$ be the in-degree and out-degree of node v_i, respectively. It is easy to obtain that

$$\sum_{i=1}^{N} k_i^- = \sum_{i=1}^{N} k_i^+ = W. \tag{1}$$

We only consider disintegration approaches against nodes in this study and assume that the attached edges are removed if one node is removed. Denote by $\hat{V} \subseteq V$ the set of nodes that are removed and denote by $\hat{G} = (V - \hat{V}, \hat{E})$ the network after the node removal. Denote by $n = |\hat{V}|$ the disintegration strength parameter. We define a disintegration strategy as $\hat{X} = [x_1, x_2, \dots, x_N]$, where $x_i = 0$ if $v_i \in \hat{V}$, otherwise $x_i = 1$. Then we obtain

$$n = N - \sum_{i=1}^{N} x_i. \tag{2}$$

The measure function of network performance is denoted by $\Gamma(G)$. We assume that if $G_1 = (V_1, E_1)$ is a subgraph of $G_2 = (V_2, E_2)$, i.e. if $V_1 \subseteq V_2$ and $E_1 \subseteq E_2$, then $\Gamma(G_1) \leq \Gamma(G_2)$. This monotonicity assumption ensures that the network performance reduces with the process of network disintegration. We define the effect of disintegration strategy as the degradation of network performance after node removal $\Phi(\hat{X}) = \Gamma(G) - \Gamma(\hat{G}) \geq 0$. Our goal is to find the optimal disintegration strategy $\hat{X}^*$, which maximizes the disintegration effect $\Phi(\hat{X})$. Therefore, we obtain the optimization model for disintegration strategy as follows

$$\max\ \Phi(\hat{X} = [x_1, x_2, \dots, x_N]) \\ \text{s.t.} \begin{cases} \sum_{i=1}^{N} x_i = N - n \\ x_i = 0 \text{ or } 1, i = 1, 2, \dots, N \end{cases}. \tag{3}$$

3. Solution based on tabu search

As the objective function Φ generally has no explicit form, we cannot solve the optimization model in Eq. (3) using traditional integer programming techniques. Actually, it can be seen as a combinatorial optimization problem. For a network with N nodes, there are C_N^n ways to pick up n nodes for removal, which grows dramatically with N and n. For example, for $N = 100$ and $n = 10$, the total number of alternative strategies will be astronomically large, corresponding to $C_{100}^{10} = 100!/(10! \times 90!) \approx 1.73 \times 10^{13}$. Therefore, traversing all possible solutions is almost impossible if the network size is large. Here, we consider solving this problem using tabu search (TS).

For most cases, tabu search is a meta-heuristic search algorithm and used as an efficient tool for solving global optimization problems [35]. It employs local search methods which take the improved one among immediate neighbors of the current solution as the next solution iteratively until any improved solution cannot be found. By introducing mutation operation, the ability of escaping the local optimum is strengthened and the ability of finding global optimum is enhanced. TS has five important parts to ensure its fundamental function: move mechanism, prohibition rule, tabu list, aspiration criterion,