Physica A 507 (2018) 219–230

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Enhancing qubit information with quantum thermal noise

Nicolas Gillard, Etienne Belin, François Chapeau-Blondeau *

Laboratoire Angevin de Recherche en Ingénierie des Systèmes (LARIS), Université d'Angers, 62 avenue Notre Dame du Lac, 49000 Angers, France

HIGHLIGHTS

- Several generic informational quantities characterizing the qubit are analyzed.
- Qubit decoherence is represented by a quantum thermal noise at arbitrary temperature.
- Nontrivial regimes of variation are reported for the informational quantities.
- They do not always degrade but can show nonmonotonic variation at increasing temperature.
- Higher noise temperatures or increased decoherence may prove beneficial informationally.

ARTICLE INFO

Article history:
Received 2 March 2018
Received in revised form 20 April 2018
Available online 17 May 2018

Keywords:
Quantum noise
Quantum information
Entropy
Stochastic resonance
Improvement by noise
Decoherence

ABSTRACT

Informational quantities characterizing the qubit are analyzed in the presence of quantum thermal noise modeling the decoherence process due to interaction with the environment represented as a heat bath at arbitrary temperature. Nontrivial regimes of variation are reported for the informational quantities, which do not necessarily degrade monotonically as the temperature of the thermal noise increases, but on the contrary can experience nonmonotonic variations where higher noise temperatures can prove more favorable. Such effects show that increased quantum decoherence does not necessarily entail poorer informational performance, and they are related to stochastic resonance or noise-enhanced efficiency in information processing.

1. Introduction

For information processing it is known that, in some specific situations, noise is not necessarily a nuisance but can sometimes prove beneficial. Such possibility has been largely explored in the context of classical (non-quantum) information processing, especially in relation to the phenomenon of stochastic resonance under its many forms [1–7]. Investigation of stochastic resonance or noise-enhanced efficiency in information processing, has been extended to the quantum domain. Early studies on quantum stochastic resonance concentrated on noise-enhanced transmission of a periodic driving [8–13]. In different information processing contexts, stochastic resonance has been shown in networks of coupled spins [14–16], or in other high-dimensional quantum systems [17–19]. For tasks of quantum state detection or discrimination stochastic resonance has been reported in [20,21], and for quantum state estimation in [22,23].

For information transmission over noisy qubit channels – which is the main theme of the present report – stochastic resonance has been addressed by several studies. Refs. [24–26] considered Pauli qubit channels, forming a special class of unital noise channels, and found that enhancement by noise is dependent on the measure of performance and does not

* Corresponding author.
E-mail address: chapeau@univ-angers.fr (F. Chapeau-Blondeau).

https://doi.org/10.1016/j.physa.2018.05.099
0378-4371/

always exist for common informational measures. Ref. [27] showed stochastic resonance in a convex combination of a phase-flip channel and an amplitude damping channel. A related effect is investigated in [28,29] under the name of superactivation, when two noisy quantum channels with zero information capacity can be used together to provide a positive capacity, with illustration with the depolarizing channel in [29].

Here in this report, we concentrate on quantum thermal noise, which is a (nonunital) noise model of great significance where the decohering environment affecting the qubits is represented as a heat bath at an arbitrary temperature. This quantum noise model has been recently analyzed for noise-enhanced detection [21] or estimation [22,23] tasks. In the present report, to further assess the possibility of noise-enhanced performance, we analyze informational quantities relevant to the qubit and based on the von Neumann entropy. Especially, among the informational quantities we shall examine are the entropy exchange, the coherent information, the quantum mutual information, the information loss, the information noise, and the Holevo information. Each of these quantities comes with a significance in relation to specific informational processes, and can serve as a measure of performance related to such informational processes. Based on the geometric representation of qubit states with Bloch vectors and on the Kraus representation of the quantum thermal noise, analytical expressions will be derived for each informational quantity. Especially, these analytical expressions will allow us to analyze the impact of the temperature of the thermal noise on the informational quantities. Nontrivial regimes of variation will be reported here for the first time for such informational measures, demonstrating that they do not necessarily degrade monotonically as the temperature of the thermal noise increases, but that on the contrary they can experience nonmonotonic variations where higher noise temperatures can prove more favorable to information transmission.

2. Qubit entropy

A qubit with two-dimensional Hilbert space $\mathcal{H}_2$ is prepared in a quantum state represented by the density operator ρ parameterized in Bloch representation as [30]

$$\rho = \frac{1}{2}\big(\mathrm{I}_2 + \vec{r}\cdot\vec{\sigma}\,\big) , \tag{1}$$

with the real 3-dimensional Bloch vector $\vec{r} \in \mathbb{R}^3$ of Euclidean norm $\|\vec{r}\,\| \le 1$, and $\vec{\sigma}$ a formal vector assembling the three 2×2 Pauli matrices $[\sigma_x, \sigma_y, \sigma_z] = \vec{\sigma}$, and I_2 the identity of $\mathcal{H}_2$.

A qubit with Bloch vector $\vec{r}$ has a density operator ρ in Eq. (1) with two eigenvalues $\lambda_\pm = (1 \pm \|\vec{r}\,\|)/2$, so that its von Neumann entropy $S(\rho) = -\,\mathrm{tr}[\rho\log(\rho)]$ results as

$$S(\rho) = h\left(\frac{1+\|\vec{r}\,\|}{2}\right) + h\left(\frac{1-\|\vec{r}\,\|}{2}\right) , \tag{2}$$

with the auxiliary function $h(u) = -u\log_2(u)$. The von Neumann entropy $S(\rho)$ of Eq. (2) is a nonnegative and monotonically decreasing function of the Bloch vector norm $\|\vec{r}\,\|$. A qubit in a pure state ρ has $\|\vec{r}\,\| = 1$ and entropy $S(\rho) = 0$. A mixed state ρ has $\|\vec{r}\,\| < 1$ and entropy $S(\rho) > 0$, which reaches the maximum $S(\rho) = 1$ when $\|\vec{r}\,\| = 0$ for the maximally mixed state $\rho = \mathrm{I}_2/2$. It results that the entropy $S(\rho)$ is interpretable as a measure of disorder or unpredictability of the quantum state ρ, with $S(\rho)$ monotonically increasing as ρ passes from a pure quantum state to the maximally mixed state $\rho = \mathrm{I}_2/2$.

We consider that the qubit in state ρ is transmitted by a noisy communication channel generally representable by a completely positive trace-preserving superoperator $\mathcal{N}(\cdot)$ implementing the input–output transformation [30,31]

$$\rho \longrightarrow \rho' = \mathcal{N}(\rho) = \sum_{k=1}^{K} \Lambda_k \rho \Lambda_k^\dagger , \tag{3}$$

characterized by the K Kraus operators Λ_k satisfying $\sum_{k=1}^{K}\Lambda_k^\dagger\Lambda_k = \mathrm{I}_2$. This is equivalent to transforming the Bloch vectors by the affine map [30,31]

$$\vec{r} \longrightarrow \vec{r}\,' = A\vec{r} + \vec{c} , \tag{4}$$

with A a 3×3 real matrix and $\vec{c}$ a real vector in $\mathbb{R}^3$. We are specifically interested in studying the impact of a quantum noise channel $\mathcal{N}(\cdot)$ very important for the qubit, which is the generalized amplitude damping noise or quantum thermal noise [30,31]. Such thermal noise, unlike other less sophisticated noise models for the qubit, lends itself to nontrivial noise effects manifested by the entropy and other useful informational measures exhibiting unusual variations, as we shall see. The quantum thermal noise [30,31] is characterized in Eq. (3) by the $K = 4$ Kraus operators

$$\Lambda_1 = \sqrt{p}\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{bmatrix}, \tag{5}$$

$$\Lambda_2 = \sqrt{p}\begin{bmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{bmatrix}, \tag{6}$$

$$\Lambda_3 = \sqrt{1-p}\begin{bmatrix} \sqrt{1-\gamma} & 0 \\ 0 & 1 \end{bmatrix}, \tag{7}$$