Physica A 506 (2018) 900–908

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Effects of speed deviation and density difference in traffic lattice hydrodynamic model with interruption

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HIGHLIGHTS

- A new traffic lattice hydrodynamic model is proposed by considering effects of speed deviation, traffic interruption probability and density difference.
- The mKdV equation is derived to describe the traffic jam.
- The effect of above three factors can improve the stability of traffic flow.

ARTICLE INFO

Article history:
Received 27 March 2018
Received in revised form 11 April 2018
Available online 12 May 2018

Keywords:
Traffic flow
mKdV equation
Speed deviation
Density difference

ABSTRACT

An extended lattice hydrodynamic model is proposed by incorporating the effects of speed deviation, traffic interruption probability and the density difference between the leading and the following lattice. The stability condition of the extended model is obtained by using the linear stability theory. Based on nonlinear analysis method, the mKdV equation is derived. Therefore, the propagation behavior of traffic jam can be described by the kink-antikink soliton solution of the mKdV equation. Numerical simulation demonstrated that the traffic flow will be more stable and it is more consistent with real traffic with the consideration of effects of speed deviation and traffic interruption probability and the density difference.

1. Introduction

With the development of urbanization, traffic flow becomes a topic issue in social problems in recent years. Traffic jam has become a serious problem, because it will result in security risks for drivers and vehicles. In order to solve the traffic problem, an increasing number of models have been proposed by scholars with different backgrounds, including car-following models [1–21], cellular automation models [22–26], macro traffic models [27–31] and lattice hydrodynamic models [32,33]. As one of traffic flow models, lattice hydrodynamic model [29] was firstly proposed by Nagatani in 1998. And he derived the mKdV equation through this model to describe traffic jam as a kink-antikink density wave without taking some factors into consideration. Later, in order to describe the traffic jam more realistically, a multitude of novel lattice hydrodynamic models [34–48] have been proposed by considering different factors such as "backward looking" effect, traffic interruption probability, driver's delay response, speed deviation, driver's aggressive characteristics, and so on.

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https://doi.org/10.1016/j.physa.2018.05.023
0378-4371/

In recently years, many researchers have taken into account the effect of traffic interruption as traffic often occurs with some probabilities for the reasons of the factors like traffic accident, pedestrian and traffic light. Tang et al. [49,50] considered the traffic interruption probabilities in the car-following model and macro model. Subsequently, Redhu et al. [51] proposed an extended lattice hydrodynamic model. Furthermore, Peng et al. [52] researched the effect in lattice hydrodynamic model. However, these models cannot reveal the traffic property results from the interruption of the current lattice, so other factors were considered. Zhang et al. [53] proposed a novel lattice hydrodynamic model with the consideration of the density difference effect. In addition, Zhou et al. [51] and Tian et al. [54] researched the influence of the density difference. However, the effect of speed deviation was not taken into consideration in these models. In view of this, an extended lattice hydrodynamic model is presented to investigate the effects of speed deviation, density difference and traffic interruption probability in this paper.

In Section 2, the extended model is formulated by incorporating the three factors. Therefore, the stability condition is derived with the linear theory in Section 3. In Section 4, the mKdV equation is obtained by using the method of nonlinear analysis. For the sake of demonstration of theoretical results, numerical simulation is carried out in Section 5. Conclusions are drawn in Section 6.

2. The extended lattice hydrodynamic model

Based on the original lattice hydrodynamic model, an extended lattice hydrodynamic model is proposed by incorporating the effects of speed deviation, traffic interruption probability and the density difference between the leading and the following lattice. Therefore, the motion equation is given as follows:

$$\partial_t(\rho_j v_j) = a\rho_o V(\rho_{j+1}) - a(1-p_j)(1+k)\rho_j v_j + \lambda\frac{\rho_j - \rho_{j+1}}{\rho_o}, \tag{1}$$

$$\partial_t \rho_j + \rho_o(\rho_j v_j - \rho_{j-1}v_{j-1}) = 0, \tag{2}$$

where λ represents the reactive coefficient to the density difference and k is the deviation degree of speed v. As $k > 0$, it means high estimate of the local speed v and it means low estimate of the local speed v when $k < 0$. Moreover, the effect of speed deviation will disappear when $k = 0$. For simplicity, the traffic interruption probability p_j is assumed to be a constant p. $V(\rho)$ is the optimal velocity function that is assumed to be:

$$V(\rho) = \frac{v_{max}}{2}[\tanh(\frac{2}{\rho_o} - \frac{\rho}{\rho_o^2} - \frac{1}{\rho_c}) + \tanh(\frac{1}{\rho_c})], \tag{3}$$

where ρ_o is the initial density, ρ_c represents the safety density and $v_{max} = 2$ is the maximal velocity. Eliminating speed v in Eqs. (1) and (2), we obtain the following density equation:

$$\begin{aligned}&\rho_j(t+2\tau) - \rho_j(t+\tau) + \tau\rho_o^2[V(\rho_{j+1}(t)) - V(\rho_j(t))] - (p-k+pk)[\rho_j(t+\tau) - \rho_j(t)]\\ &\quad + \tau^2\lambda(2\rho_j - \rho_{j+1} - \rho_{j-1}) = 0.\end{aligned} \tag{4}$$

3. Linear stability analysis

The linear stability analysis is performed to research the effects of speed deviation, traffic interruption probability and the density difference on traffic stability. It is obvious that the uniform traffic flow with constant density ρ_o and constant optimal velocity V_ρ is the steady state solution for Eqs. (1) and (2), given as:

$$\rho_j(t) = \rho_0,\ v_j(t) = V(\rho), \tag{5}$$

y_j is a small deviation from the steady-state flow on site j,

$$\rho_j(t) = \rho_o + y_j(t). \tag{6}$$

Inserting Eq. (6) into Eq. (4), we get the following equation:

$$\begin{aligned}&y_j(t+2\tau) - y_j(t+\tau) + \tau\rho_o^2[V(\rho_o)'(y_{j+1}(t) - y_j(t))] - (p-k+pk)[\rho_o + y_j(t+\tau)]\\ &\quad + \tau^2\lambda[2y_j(t) - y_{j+1}(t) - y_{j-1}(t)],\end{aligned} \tag{7}$$

Expanding $y_j(t) = exp(ikj + zt)$, it reads:

$$e^{2z\tau} - e^{z\tau} + \tau\rho_o^2[V(\rho_o)'(e^{ik} - 1)] + (p-k+pk)(1-e^{z\tau}) - \tau^2\lambda(ik)^2, \tag{8}$$

where $V(\rho_o)' = \frac{dV_\rho}{d\rho|\rho=\rho_o}$. Let $z = z_1(ik) + z_2(ik)^2 + \cdots$, then the first-order and second-order terms of ik are:

$$z_1 = -\frac{\rho_o^2 V(\rho_o)'}{(1-p)(1+k)}, \tag{9}$$